Mathematical Economics

LINEAR PROGRAMMING PROBLEMS

Presented to:

presented by:

Dr. T Sudha

S.SUshmitha

Associate professor 1st MAEconomics

Dept. of Economics

Bharathidasan University

LPP

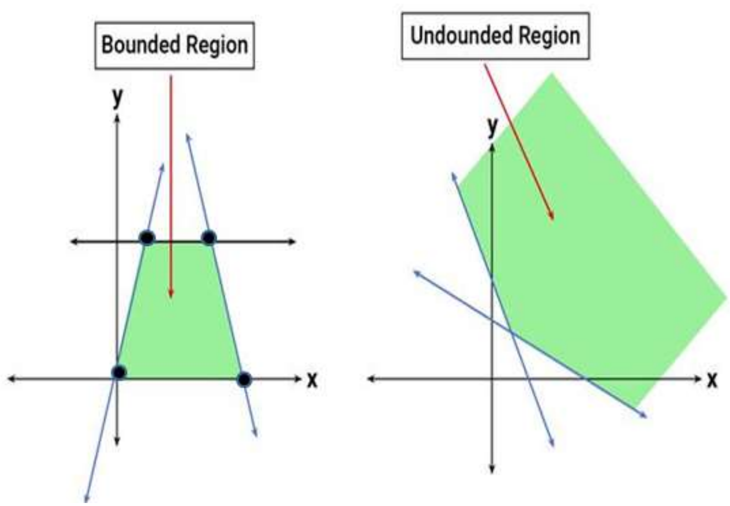
A linear programming problem (LPP) is a set of linear equations that are used to find the value of variables to optimize the objective functions.

Common terminologies used in Linear Programming:

1. **Decision variables**: x and y are the decision variables in an objective function.
2. **Non-negative constraints:** These conditions $x, y \geq 0$ are called non-negative constraints.
3. **Constraints:** The linear inequalities or equations on the variables of a linear programming problem are called constraints.
4. **Optimisation problem:** This is a problem which seeks to minimise or maximise a linear function subject to certain constraints as found by a set of linear inequalities. LPP are special types of optimization problems.
5. **Feasible region:** The region which is common to all the constraints including the non-negative constraints is referred to as feasible region.
6. **Infeasible region:** It is the region other than the feasible region.
7. **Feasible solutions:** Points within and on the boundary of a feasible region denote feasible solutions of the constraints.
8. **Infeasible solutions:** Any point which is outside the feasible region is known as infeasible solution.
9. **Optimal solution:** If any point in the feasible region gives minimum or maximum value of the objective function, it is called an optimal solution.

IMPORTANT THEOREMS

- **Theorem 1:** Let R be the feasible region for an LPP and Z be the objective function. The optimal value of Z must occur at the corner point of the feasible region.
- **Theorem 2:** Let R be the feasible region. If the R is bounded, then the objective function Z, has both maximum and minimum value in region R. And both of them occur at the corner points of R.
- If R is unbounded , then a minimum or maximum value of the objective function may not exist. If it exists, it should come at a corner point of R (by theorem 1)



Methods to solve Linear Programming Problems

There are different methods to solve any Linear Programming Problem.

1. *Graphical Method*
2. *Simplex Method*
3. *North West Corner Method*
4. *Least Square Methods*

For now, we will discuss only the Graphical method to solve an LPP.

Linear Programming by Graphical Method

If there are two decision variables in a linear programming problem then the graphical method can be used to solve such a problem easily.

PROBLEM

Q] Maximize $Z = 2x + 5y$. The constraints are $x + 4y \leq 24$, $3x + y \leq 21$ and $x + y \leq 9$ where, $x \geq 0$ and $y \geq 0$.

Step 1: Write all inequality constraints in the form of equations.

$$x + 4y = 24$$

$$3x + y = 21$$

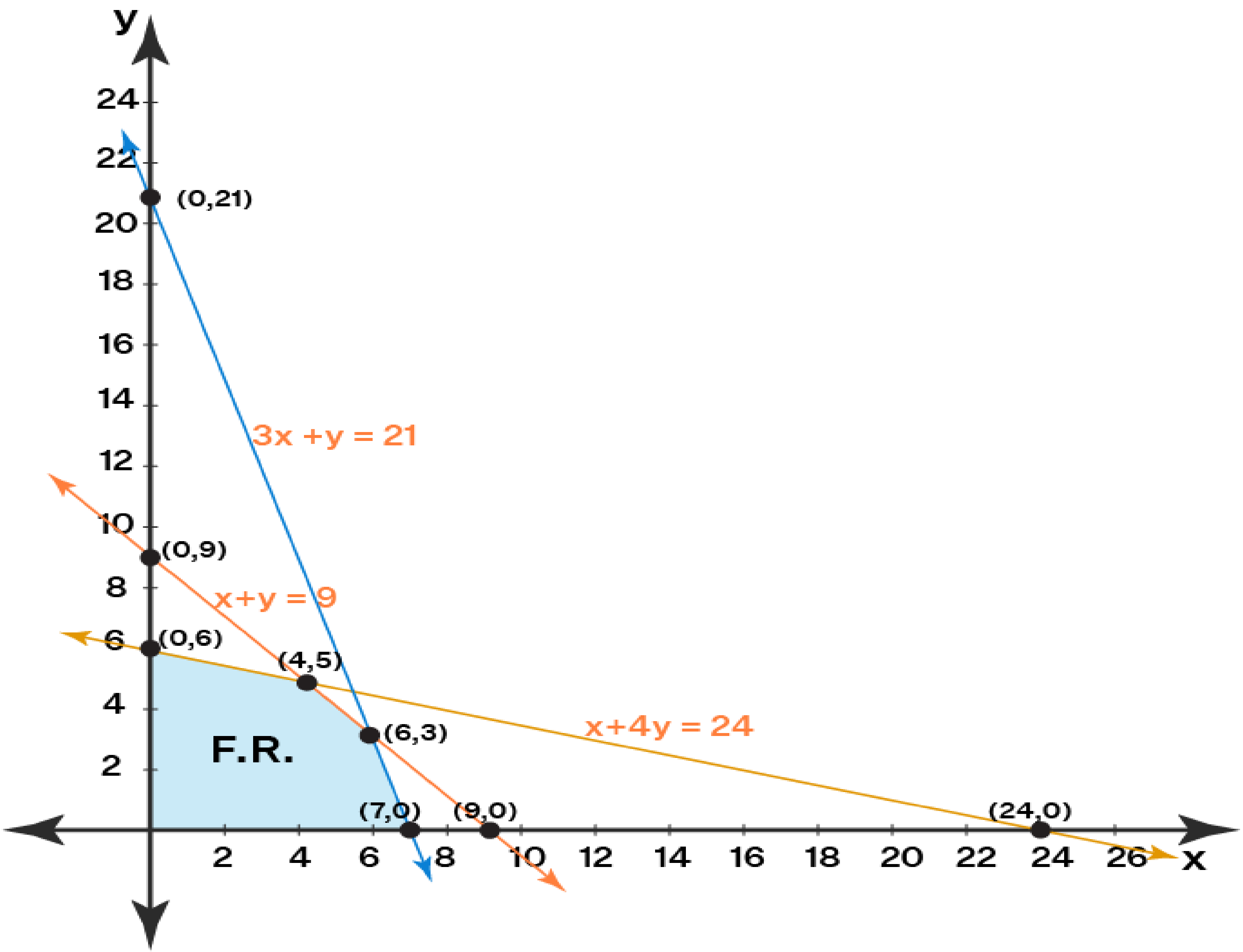
$$x + y = 9$$

Step 2: Plot these lines on a graph by identifying test points.

x + 4y = 24 is a line passing through (0, 6) and (24, 0). [By substituting x = 0 the point (0, 6) is obtained. Similarly, when y = 0 the point (24, 0) is determined.]

3x + y = 21 passes through (0, 21) and (7, 0).

x + y = 9 passes through (9, 0) and (0, 9).


y
x
(0,21)
3x +y = 21
(0,9)
x+y = 9
(0,6)
(4,5)
(6,3)
x+4y = 24
F.R.
(7,0)
(9,0)
(24,0)
24
22
20
18
16
14
12
10
8
6
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2
2
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26

Step 3: Identify the feasible region. The feasible region can be defined as the area that is bounded by a set of coordinates that can satisfy some particular system of inequalities.

- Any point that lies on or below the line $x + 4y = 24$ will satisfy the constraint $x + 4y \leq 24$.
- Similarly, a point that lies on or below $3x + y = 21$ satisfies $3x + y \leq 21$.
- Also, a point lying on or below the line $x + y = 9$ satisfies $x + y \leq 9$.
- The feasible region is represented by OABCD as it satisfies all the above-mentioned three restrictions.

Step 4: Determine the coordinates of the corner points. The corner points are the vertices of the feasible region.

O = (0, 0)

A = (7, 0)

B = (6, 3) (B is the intersection of the two lines $3x + y = 21$ and $x + y = 9$. Thus, by substituting $y = 9 - x$ in $3x + y = 21$ we can determine the point of intersection.)

C = (4, 5) (formed by the intersection of $x + 4y = 24$ and $x + y = 9$)

D = (0, 6)

Step 5: Substitute each corner point in the objective function. The point that gives the **greatest (maximizing)** or **smallest (minimizing)** value of the objective function will be the optimal point.

CORNER POINT	Z = 2x +5y
O = (0, 0)	0
A = (7, 0)	14
B = (6, 3)	27
C = (4, 5)	33
D = (0, 6)	30

33 is the maximum value of Z and it occurs at C.

Thus, the solution is **x** = 4 and **y** = 5.

References

WEBSITES

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Welcome

Mathematical Economics

Linear Programming, Importance, Characteristics.

Presented to
E.Arthi
I M A Economics

Presented to
Dr. T. Sudha
Department of Economics

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- Introduction
- Characteristics
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INTRODUCTION:

- *Linear programming is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value.*

Characteristics of Linear Programming :

- A decision amongst alternative courses of action is required.
- The decision is represented in the model by decision variables.
- The problem encompasses a goal, expressed as an objective function, that the decision maker wants to achieve.

IMPORTANCE:

- **Optimization:**

  Linear programming allows for the optimization of resources, costs, and profits, leading to better decision-making and improved efficiency.

- **Resource Allocation:**

  It helps in allocating scarce resources efficiently, ensuring maximum utilization and minimal wastage.

- ***Strategic Planning:***

Businesses can use linear programming to develop strategic plans, considering various constraints and objectives.

- ***Cost Reduction:***

By optimizing processes and resource allocation, linear programming can lead to cost reduction and increased profitability.

- ***Risk Management:***

In finance, it helps in managing risks associated with investments and portfolio management.

CONCLUSION:

- Linear Programming is a versatile tool with real-world applications across various domains.
- Its ability to solve complex optimization problems makes it invaluable for businesses and organizations seeking efficient, cost-effective solutions.

Thank you