Mathematical Economics

Topic:Cramer's Rule

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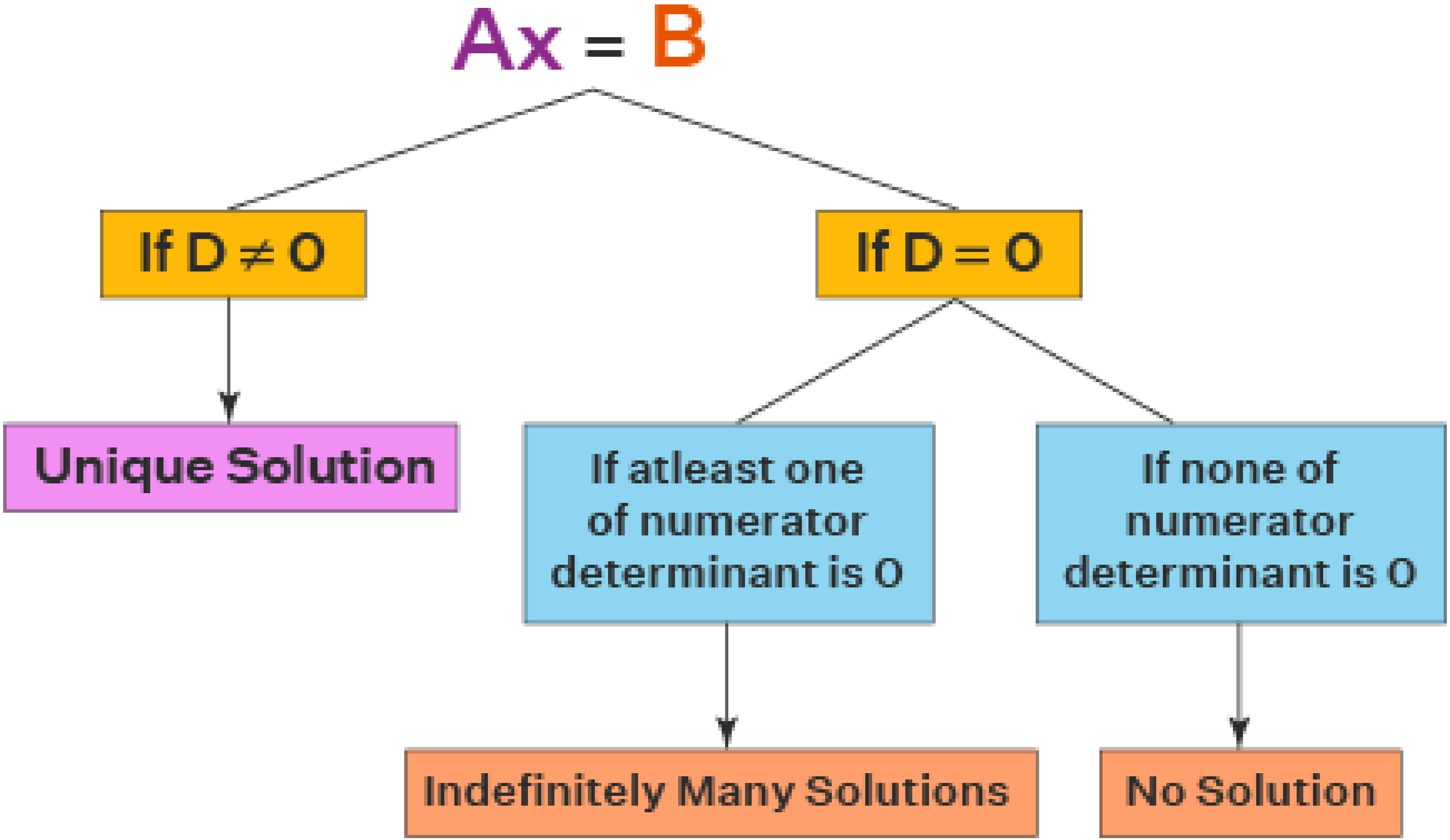
Contents

Introduction

- Cramer's Rule is a method for solving systems of linear equations by expressing each variable in terms of determinants.
- Named after Swiss mathematician Gabriel Cramer(1704-1752), this technique is applicable when the coefficient matrix of the system is non-singular.
- The rule provides a formula for each variable using determinants, offering an analytical approach to solving linear systems.
- However, it may become impractical for larger systems due to the computational complexity of determinants.

Cramer's Rule

- ❑Cramer's rule is one of the methods used to solve a system of equations. This rule involves determinants. i.e., the values of the variables in the system are found with the help of determinants.
- ❑ Let us consider a system of equations in n variables x_1, x_2, x_3, ..., x_n written in the matrix form AX = B where,
- ✓A = the coefficient matrix which is a square matrix.
- ✓X = the column matrix with variables.
- ✓B = the column matrix with the constants.


Ax = B
If D ≠ 0
If D = 0
Unique Solution
If atleast one of numerator determinant is 0
If none of numerator determinant is 0
Indefinitely Many Solutions
No Solution

Conditions

- From the above chart and explanation, it is very clear that Cramer's rule is NOT applicable when D = 0. i.e., when the determinant of the coefficient matrix is 0, we cannot find the solution of the system of equations using Cramer's rule.
- In this case, we can find the solution (if any) by using Gauss Jordan Method.
- Thus, Cramer's rule is used to find the solution of a system only when the system has a unique solution.

Examples

- Solve the Determinant 2*2 Matrix

  1) $5x-2y+16=0$, $x+3y-7=0$

- Solve the Determinant 3*3 Matrix

  1) $3x+3y-z=11$, $2x-y+2z=9$, $4x+3y+2z=25$

Conclusion

- In economics, Cramer's Rule can be employed to solve systems of linear equations that often arise in economic modeling.
- Economic models frequently involve multiple variables and equations, representing relationships between different economic factors.

Thank You

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Mathematical Economics

Topic : Properties of Determinants

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Content :

→ Properties of Determinants :

- Property of interchange
- Sign property
- Zero property
- Multiplication property
- Sum property
- Property of Invariance
- Triangular property

What Are the Properties of Determinants?

The features of determinants aid in quickly calculating the value of a determinant with the fewest steps and calculations possible. The following are the 7 most important properties of determinants.

1. Property of Interchange:

When the rows or columns of a determinant are swapped, the determinant’s value remains unchanged.
Det(A) = Det(A’)

As a result of this property, if the rows and columns of the matrix are swapped, the matrix is transposed, and the determinant value and the determinant of the transposition are both equal.

A =
$$\begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix}$$

A′ =
$$\begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix}$$

2. Sign Property:

If any two rows or columns are swapped,
the sign of the determinant’s value

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

$R1 \leftrightarrow R3$

$$B = \begin{bmatrix} 1 & 5 & -7 \\ 6 & 0 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

If the row or column is swapped once, the determinant’s value changes the sign. To obtain matrix B, the first row of matrix A has been swapped with the third row, and we have Det(A) = - Det(A) (B). If the determinant’s value is D and the rows or columns are swapped n times, the new determinant’s value is (-1)nD.

3. Zero Property:

If the elements in any two rows or columns are the same, the determinant’s value is zero.

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 1 & 5 & 7 \\ 2 & -3 & 5 \end{bmatrix}$$

The items in the first and second rows are identical in this case. As a result, the determinant’s value is zero.

Det(A) = 0

4. Multiplication Property:

If each of the elements of a given row or column is k times the earlier value of the determinant, the deciding value becomes k times the earlier value of the determinant. If each element of a given row or column is multiplied by a constant k, the determining va omes s the earlier value of the determinant.

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 6 & 0 & 4 \\ 2 & -3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 5 & 7 \\ 24 & 0 & 16 \\ 2 & -3 & 5 \end{bmatrix}$$

Det(B) = k× Det(A)

The second row of A is multiplied with a constant k. Here, k = 4.

The second row’s elements are multiplied by a constant k, and the determinant value is multiplied by the same constant. This characteristic aids in the extraction of a common factor from each determinant row or column. In addition, the value of the determinant is 0 if the corresponding elements of any two rows or columns are equal.

5. Sum Property:

The determinant can be expressed as a sum of two or more determinants if a few items of a row or column are expressed as a sum of words.

6. Property of Invariance:

When each element of a determinant’s row and column is multiplied by the equimultiples of the elements of another determinant’s row or column, the determinant’s value remains unchanged. This can be stated as a formula as follows:

$$R_i \to R_i + \alpha R_j + \beta R_k \text{ or } C_i \to C_i + \alpha C_j + \beta C_k$$

7. Triangular Property:

The value of the determinant is equal to the product of the components of the diagonal of the matrix if the elements above and below [illegible]l are both equal to zero.

$$\begin{vmatrix} a1 & a2 & a3 \\ 0 & b1 & b3 \\ 0 & 0 & c3 \end{vmatrix}$$

=

$$\begin{vmatrix} a1 & 0 & 0 \\ a2 & b1 & 0 \\ a3 & b2 & c3 \end{vmatrix}$$

= a1.b2.c3


Thank you