BHARATHIDASAN UNIVERSITY

Tiruchirappalli- 620024

Tamil Nadu, India.

Programme: M.Sc. Statistics

Course Title: Statistical Quality Control

Course Code: 23ST08CC

Unit-II

Process Capability Analysis

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UNIT-II

PROCESS CAPABILITY ANALYSIS

Process Capability Analysis

Statistical technique can be helpful throughout the product cycle including development activates prior to manufacturing, in quantifying process variability in analysing this variability relative to product requirements or specifications, and in assisting development and manufacturing in eliminating or greatly reducing this variability. This general activity is called Process capability analysis.

Process Capability

Process capability refers to the uniformly of the process. Obviously the variability of critical to quality characteristics in the process is a measure of the uniformity of output. There are two ways to think of this variability:

- The nature or inherent variability in a critical to quality characteristics at a specified time: that is, ‘instantaneous’ variability.
- The variability in a critical – to- quality characteristic over time.

It is customary to take the six sigma spread in the distribution of the product quality characteristic as a measure of process capability a process for which the quality characteristic has a normal distribution. With mean μ and standard deviation σ. The upper and lower natural tolerance limits of the process fall at $\mu+3\sigma$ and $\mu-3\sigma$.respectively that is,

$$UNTL= \mu+3\sigma$$
$$LNTL= \mu-3\sigma$$

For a normal distribution the normal tolerance limits include 99.73% of the variable or put another way, only 0.27% of the process output will fall outside the natural tolerance limits. Two points should be remembered:

- 0.27% outside the natural tolerances sounds small, but this corresponds to 2700 non conforming parts per million.
- If the distribution of the process output is non normal then the percentage of output failing outside $\mu\pm3\sigma$ may differ considerably from 0.27%

The define process capability analysis as a formal study to estimate process capability. The estimate of the process capability may be in the form of the probability distribution having specified shapes, center (mean), and spread (standard deviation).

A process capability study usually measure functional parameters or critical to quality characteristics on the product, not the process itself When the analyst can directly observe the process and can control or monitor the data collection activity, the study is a true process capability study because by controlling the data collection and knowing the time sequence of the data, inferences can be made about the stability of the process over time.

However, when we have available only sample units of product, perhaps obtained from the supplier, and there is no direct observation of the process or time history of production, then the study is more properly called product characterisation. In a product characterisation study we can only estimate the distribution of the product quality characteristic or the process yield (fraction conforming to specifications): we can say nothing about the dynamic behaviour of the process or its state of statistical control.

Process Capability Analysis Using a Histogram or a Probability Plot

Process capability analysis is a vital part of an overall quality improvement program. Among the major uses of data from a process capability analysis are the following:

- Predicting how well the process will hold the tolerances
- Assisting product developers/designers in selecting or modifying a process
- Assisting in establishing an interval between sampling for process monitoring
- Specifying performance requirements for new equipment
- Selecting between competing suppliers and other aspects of supply chain management
- Planning the sequence of production process when there is an interactive effect of processes on tolerances
- Reducing the variability in a process

Three primary techniques are used in process capability analysis; histogram or probability plots, control charts, and designed experiments. We will discuss and illustrate each of these methods in the next three sections.

Process Capability Ratio (PCR)

Capability ratios (Cp/Pp) describe the variability of a process relative to the specification limits. A capability ratio is a unit-less value describing the ratio of process distribution spread to specification limits spread.

- A value of less than 1 is unacceptable, with values greater than 1.33 (1.25 for one-sided specification limits) widely accepted as the minimum acceptable value, and
- A values greater than 1.50 (1.45 for one-sided specification limits) for critical parameters *(Montgomery, 2012)*.
- The higher the value, the more capable the process of meeting specifications. A value of 2 or higher is required to achieve Six Sigma capability which is defined as the process mean not closer than six standard deviations from the nearest specification limit.

All of the indices assume a normally distributed process quality characteristic with the parameters specified by the process mean and sigma. The process sigma is either the short-term or long-term sigma estimate.

Indices computed using the short-term sigma estimate are called Cp indices (Cp, Cpl, Cpu, Cpk, Cpm). While those using long-term sigma estimate are called Pp indices (Pp, Ppl, Ppu, Ppk, Ppm). If the Cp indices are much smaller than the Pp indices, it indicates that there are improvements you could make by eliminating shifts and drifts in the process mean.

Assumptions:

- There are two critical assumptions to consider when performing process capability analysis with continuous data, namely
  - Process is in statistical control
  - The distribution of the process considered is normal
- If these assumptions are not met, the resulting statistics mat be highly unreliable.
- In later modules we will discuss capability analysis for non normal data.

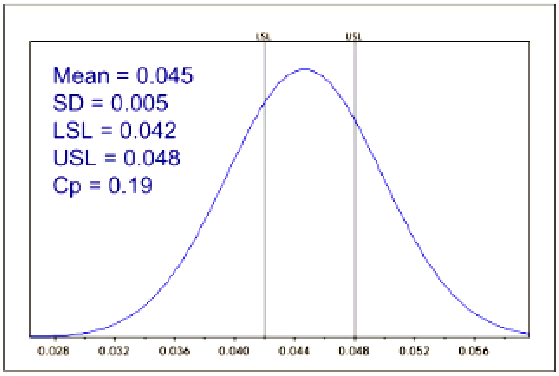
Various indices measure how the process is performing against the specification limits:

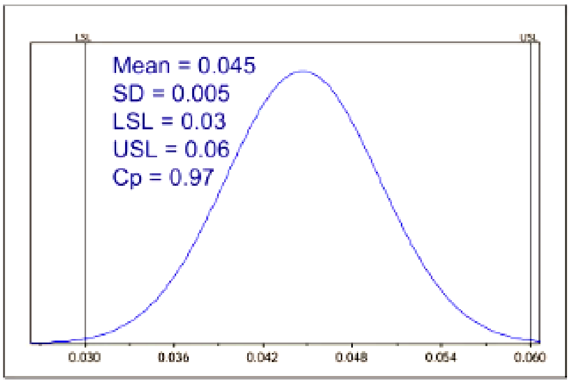
Index	Purpose
Cp/Pp	Estimates the capability of a process if the process mean were to be centered between the specification limits. Note: If the process mean is not centered between the specification limits the value is only the potential capability, and you should not report it as the capability of the process.
Cpl/Ppl	Estimates the capability of a process to meet the lower specification limit. Defined as how close the process mean is to the lower specification limit.
Cpu/Ppu	Estimates the capability of a process to meet the upper specification limit. Defined as how close the process mean is to the upper specification limit.
Cpk/Ppk	Estimates the capability of a process, considering that the process mean may not be centered between the specification limits. Defined as the lesser of Cpl and Cpu. Note: If Cpk is equal to Cp, then the process is centered at the midpoint of the specification limits. The magnitude of Cpk relative to Cp is a measure of how off center the process is and the potential improvement possible by centering the process.
Cpm/Ppm	Estimates the capability of a process, and is dependent on the deviation of the process mean from the target. Note: Cpm increases as the process mean moves towards the target. Cpm, Cpk, and Cp all coincide when the target is the center of the specification limits and the process mean is centered.

C_p:

- Approximately 99.7% of the data from a normal distribution is contained between ±3σ.
- If the process is in control and the distribution is well within the specification limits then the difference between the upper specification(U) and lower specification (L) should be larger than 6σ.
- If the specifications are larger than 6σ, the ratio will be less than 1.
- If $C_p$is greater than 1 then the process has the potential to meet specifications as lang as the mean is cantered.

$$C_p = \frac{Upper\ Spec - LowerSpec}{6\sigma}$$

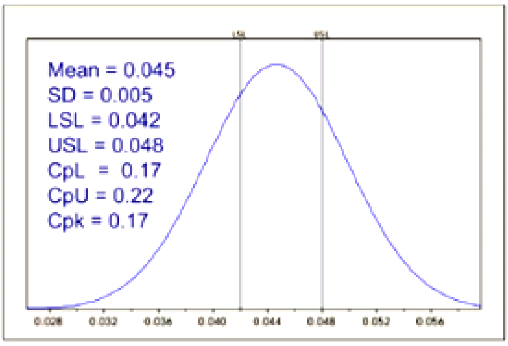


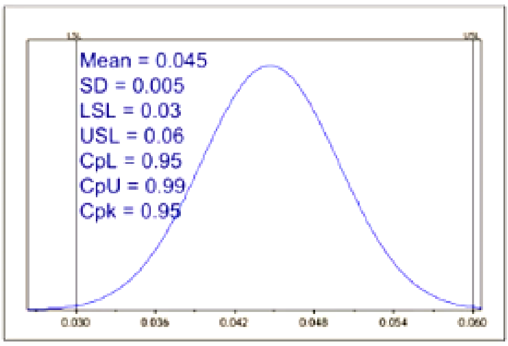


C_{pk}:

- C_{pk} is an process capability index that assesses how close the process mean is from the specification limit
- If the process is in control and the distribution is well within the specification limits then the difference between the Upper specification (U) and then mean or the difference between the lower specification (L) and the mean should be larger than then 3σ.
- If C_{pk} is greater than 1 then the process mean is sufficiently far from the specification limit.

$$C_{pk} = \frac{\text{Upper Spec} - \bar{\bar{X}}}{3\sigma} \quad \text{and} \quad C_{pk} = \frac{\bar{\bar{X}} - \text{Lower Spec}}{3\sigma} \quad \text{and} \quad C_{pk} = \text{mini}(C_{pU}, C_{pL})$$





Basic Concepts of Gauge Capability

The capability of the measurement system is an important aspect of much quality and process improvement activates. Generally, in any activity involving measurements and some of the observed variability will result from the measurement system that is used. The measurement system will consist (minimally) of an instrument or gauge, and it often has other components, such as the operator(s) that uses it and the conditions or different points in time under which the instrument is used. The purpose of most measurement system capability studies is to:

- Determine how much of the total observed variability is due to the gauge or instrument
- Isolate the components of variability in the measurement system
- Assess whether the instrument or gauge is capable (that is, is it suitable for the intended application

The two R's of measurement systems capability:

- Repeatability (do we get the same observed value if we measure the same unit several times under identical conditions) and
- Reproducibility how much difference in observed values do we experience when units are measured under different conditions, such as different operators, time, periods, and so forth.

Linearity

The linearity of a measurement system reflects the differences in observed accuracy and/or precision experienced over the range of measurements made by the system. A simple linear regression model is often used to describe this feature. Problems with linearity are often the result of calibration and maintenance issues.

Stability

The Stability or different levels of variability in different operating regimes can result from warm-up effects, environmental factors, inconsistent operator performance and inadequate standard operating procedure. Bias reflects the difference between observed measurements and a "true" value obtained from a master or gold standard, or from a different measurement technique known to produce accurate values

Because excessive measurement variability becomes part of overall product variability it also negatively impacts many other process improvement activities, such as leading to larger in sample size in comparative or observational studies, more replication in designed experiments aimed at process improvement and more extensive product testing.

Measurement Systems Analysis (MSA)

To introduce some of the basis ideas of measurement systems analysis (MSA) consider a simple but reasonable model for measurement system capability studies

$$y = x + \varepsilon$$

where,

- y is the total observed measurement,
- x is the true value of the measurement on a unit of product, and
- ε is the measurement error.

Assume that x and ε are normally and independently distributed random variables with mean μ and 0 and variances $(\sigma_p^2)\ and\ (\sigma_{gauge}^2)$ respectively. The variance of the total observed measurement, y, is then

$$\sigma_{total}^2 = \sigma_p^2 + \sigma_{gauge}^2 \rightarrow (1)$$

Control charts and other statistical methods can be used to separate these components of variance, as well as to give an assessment of gauge capability.

Precision to Tolerance (PIT) Ratio

It is a fairly common (but not necessarily good) practice to compare the estimate of gauge capability to the width of the specifications or the tolerance band (USL-LSL) for the part that is being measured. The ratio of $k\hat{\sigma}_{gauge}$ to the total tolerance band is often called the precision to tolerance (PIT) ratio;

$$PIT = \frac{k\hat{\sigma}_{gauge}}{USL - LSL} \rightarrow (2)$$

A gauge must be sufficiently capable to measure product accurately enough and precisely enough to that the analyst can make the correct decision. This may not necessarily require that PIT≤ 0.1.

Uses the data from the gauge capability experiment to estimate the variance components associated with total observed variability from the actual sample measurements.

Ratio of measurement system variability to total variability;

$$\rho_M = \frac{\sigma_{gauge}^2}{\hat{\sigma}_{total}^2}$$

Obviously, $\rho_P = 1 - \rho_M$,

$$\grave{\rho}_M = \frac{\hat{\sigma}_{gauge}^2}{\hat{\sigma}_{total}^2}$$

Example:

We calculate s=3.17. this is an estimate of the standard deviation of total variability, including both product variability and gauge variability.

$$\hat{\sigma}_{total}^2 = s^2 = (3.17)^2 = 10.05$$

Since the eqn (1) we have

$$\hat{\sigma}_{total}^2 = \sigma_p^2 + \sigma_{gauge}^2$$

And because we have an estimate of $\hat{\sigma}_{gauge}^2 = (0.887)^2 = 0.79.$ we can about an estimate of σ_p^2 as

$$\hat{\sigma}_p^2 = \hat{\sigma}_{total}^2 - \hat{\sigma}_{gauge}^2 = 10.05 - 0.79 = 9.26$$

Therefore, an estimate of the standard deviation of the product characteristic is

$$\hat{\sigma}_p = \sqrt{9.26} = 3.04$$

There are other measures of gauge capability that have been proposed. One of these is the ratio of process (part) variability to total variability;

$$\rho_P = \frac{\hat{\sigma}_p^{\ 2}}{\hat{\sigma}_{total}^2}$$

And another is the ratio of measurement system variability to total variability;

$$\rho_M = \frac{\sigma_{gauge}^2}{\hat{\sigma}_{total}^2}$$

Obviously, $\rho_P = 1 - \rho_M$,

$$\grave{\rho}_M = \frac{\hat{\sigma}_{gauge}^2}{\hat{\sigma}_{total}^2} = \frac{0.79}{10.05} = 0.0786$$

Thus the variance of the measuring instrument contributes about 7.86% of the total observed variance of the measurements.

Cumulative Sum Chart (CUSUM Chart)

A cumulative sum chart (CUSUM) is a type of control chart used to detect the deviation of the individual values or subgroup mean from the adjusted target value. In other words, monitor the deviation from the target value. CUSUM chart is an alternative to Shewhart control charts.

- The CUSUM charts can be represented in the visual method, i.e., V-mask. This method was introduced by Barnard in 1959 to check whether a process is out of control or not. Still, the generally tabular (algorithmic) method will be used to monitor the process.
- Unlike other standard control charts, all previous measurements for CUSUM charts are included in the calculation for the latest plot. But, establishing and maintaining the CUSUM is difficult.

The basic advantage of the CUSUM chart is that it is more sensitive to the small shift of the process means when compared to the Shewhart charts (Individuals I-MR or X bar charts)."

The cumulative sum chart and the exponentially weighted moving average (EWMA) charts also monitor the mean of the process, but the basic difference is unlike X bar charts. They consider the previous value means at each point. Moreover, these charts are considered a reliable estimate when the correct standard deviation exists.

Advantages of the CUSUM chart

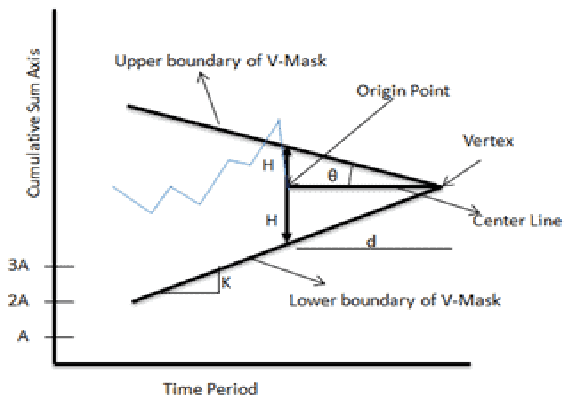
- CUSUM charts are the best way to detect the small shifts in process mean, especially 0.5 to 2 SD from the target mean.
- It is easy to identify the shifts in the process visually.

Disadvantages of the CUSUM chart

- Establishing and maintaining CUSUM charts is more complicated.
- CUSUM charts are slower in detecting large process mean shifts.
- Since CUSUMs are correlated, it is tough to interpret the patterns.

V-Mask Method

V-mask looks like a sideways V. The V-mask chart checks whether each marked sample falls within the boundaries of the V-mark. If any point falls outside of the control limit, that indicates a signal mean shift in the process. When each sample is plotted, the V-mask may have shifted to the right. The below graph shows the V-mask and related formulas.



The behaviour of the V-Mask is determined by the distance k (which is the slope of the lower arm) and the rise distance h. The team could also specify d and the vertex angle (or, as is more common in the literature, q = 1/2 the vertex angle). For an alpha and beta design approach, we must specify

- α, the probability of concluding that a shift in the process has occurred, when in fact, it did not.
- β, the probability of not detecting that a shift in the process mean has, in fact, occurred.
- δ (delta), the detection level for a shift in the process mean, expressed as a multiple of the standard deviation of the data points.

$$d = \left(\frac{2}{\delta^2}\right) In \left(\frac{1-\beta}{\alpha}\right)$$

$$\theta = tan^{-1}\left(\frac{\delta}{2A}\right)$$

$$\delta = \frac{\Delta}{\sigma_{\bar{x}}}$$

Some of the disadvantages and problems associated with this scheme are as follows:

- The head start feature, which is very useful in practices, cannot be implemented with the v-mask.
- It is sometimes difficult to determine how far backward the arms of the v-mask should extend, thereby making interpretation difficult for the practitioner:
- Perhaps the biggest problem with the v-mask is the ambiguity associated with α and β in the Johnson design procedure.

Tabular Method

The Tabular method is an easier method than the V-Mask method. Here are the steps to make a CUSUM chart.

- First of all, estimate the standard deviation of the data from the moving range control chart $\sigma = \bar{R}/d_2$.
- Calculate the reference value or allowable slack since the CUSUM chart monitors the small shifts. Generally, 0.5 to 1 sigma will be considered. K= 0.5 σ.
- Compute decision interval H, generally ± 4 σ will be considered (some place ± 5 σ also be used).
- Calculate the upper and lower CUSUM values for each individual i value.
- Upper CUSUM (UC_i)= Max[0, UC_{i-1}+x_i – Target value-k).
- Lower CUSUM (LC_i)= Min[0, LC_{i-1}+x_i – Target value +k).
- Draw all UC_i & LC_i values in the graph and also draw decision intervals (UCL and LCL).
- Check if any of the UC_i values are above the UCL and any of the LC_i values are below the LCL.
- Finally, take necessary action to eliminate the special causes if any of the points are out of control limits.

Calculate the plotting position

To find the plotting position for the new point in a cumulative sum chart, you can use the following formula.

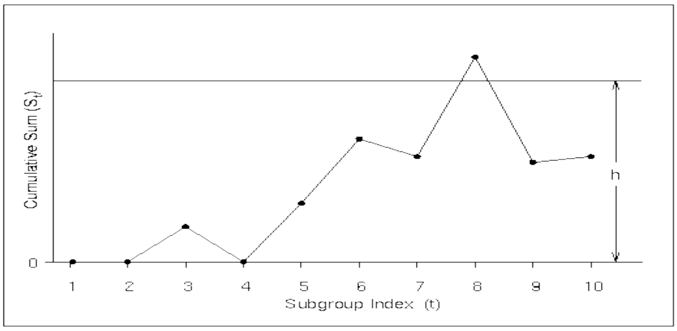
$$S_n = k + \frac{1}{\sigma_x}\sum_{i=1}^{n}(x_i - T)$$

where:

- S_n = Plotting Position
- k = reference value or the center line of the cumulative sum chart (if provided)
- x_i = The new point you want to plot
- T=Target value
- σ_x = Std Dev

Defining the Decision Interval for a One-Sided CUSUM Scheme

The height of the decision interval is *h*, expressed as a multiple of the standard error of the subgroup mean. You can specify *h* with the H= option in the X bar CHART statement or with the variable data set. The decision interval is displayed as a horizontal line on the CUSUM chart, as illustrated as below



Decision Interval

Economic model

Economic model is a framework for the economic design of control charts. It is closely linked to a sampling policy, which determines a method of sampling. A sampling policy depends or the production type. Economic design of control charts is carried out on the basis of a loss- cost function optimisation. The sampling policy changes, the results of optimisation are not valid anymore.

The issue of quality control prevent to produce nonconforming products. A fraction of nonconforming, which are manufactured on a particular work system, has a very important impact on design.

The goal of economic design of control charts is to obtain the minimum costs of the production process. With quality control we want to achieve that only conforming products reach customers. Within the proposed model, the costs related to quality control should be minimised. In the proposed economic model, the following costs are included:

- Costs of quality inspection
- Costs of false alarm
- Costs of manufacturing nonconforming products
- Costs are location and repairing assignable causes of the process.

The economical model is the frame work for loss-cost function derivations. Loss-cost function is a powerful tool due to its simplicity of problem representation in the engineering area. Loss-cost functions are presented. For the case of simplicity of notation the loss-cost function will be named loss function.

Relative precision

Relative precision is defined as a ratio of the precision of a given measurement and the value of the measurement itself. Thus, if d is a measured distance, and s_d is the standard deviation of the measurement, then the Relative precision is s_d/d. It is expressed as percentage or a fractional ratio such as 1/500 or by parts per million (ppm).

Multivariate Quality Control Charts

Multivariate quality control charts are used to monitor two or more related variables simultaneously. They are helpful for analyzing process parameters jointly with a relatively low rate of false alarms. Using these charts minimize the need for developing and managing more univariate control charts. On the other hand, their main drawback is the fact that their calculations are more complex than the ones for univariate control charts due to the use of matrix algebra.

Introduction to T^2 Control Chart

In 1947, Harold Hotelling introduced a statistic which allowed multivariate observations to be plotted on a single chart. This statistic is now called Hotelling's T^2 statistic. The statistic combines information from the mean as well as the dispersion of more than one variable. The calculations, which include some matrix algebra, are more difficult than those of "normal" control charts. This was a barrier to using multivariate control charts until software that could perform the calculations came along.

The T^2 control chart is used to detect shifts in the mean of more than one interrelated variable. The data can be in subgroups (like the X-R control chart) or the data can be individual observations (like in the X-mR control charts). The T^2 control chart shows an out of control point for batch 13. That out of control point does correspond to the low point on the scatter diagram above. Look at the T^2 control chart in the Figure. One of the first things you may notice about this control chart is that the value of T^2 has no resemblance to the original pH and viscosity data. So, looking at the value of T^2 really tells you nothing about the original data.

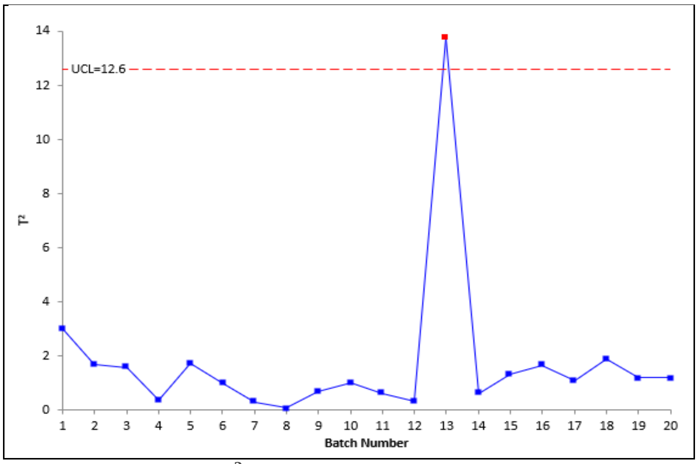


Figure - T^2 Control for pH and Viscosity

Multivariate attribute control chart using Mahalanobis D^2 statistic

There are many situations when inspection classifies the products into several categories of nonconformities. A control scheme is required to exercise simultaneous control of all the categories. With the existing tools one can apply several proportion defective (p) charts – one for each category of defect. However, this will be equivalent to testing several equality of proportion defective hypotheses independently. The problem here is of the type H_0: pi = p where pi ~ multinomial (ni, p). So it is easy to check that in the several p charts case the type I error and consequently the power of the test will suffer a distortion. To take care of the above situation a simultaneous test of pi = p was thought of. There are two ways of looking at the data matrix. One may be interested in comparing the columns of the data matrix, i.e. the variables. This leads to techniques known as R-techniques, so called because the correlation matrix R plays an important role in this approach.

Principal component analysis, factor analysis, canonical correlation analysis fall under this group of techniques. But in the present case the interest is of comparing rows of the data matrix, i.e. the different objects that are time points here. This leads to techniques such as discriminant analysis, cluster analysis, multidimensional scaling which are known as Q techniques. The Mahalanobis' concept of distance between objects plays an important role in all these Q techniques [1,4].

The Mahalanobis distance

For a data matrix whose columns represent variables and the rows represent the objects, a natural way to compare two rows X_r and X_s is to look at the Euclidean distance between them.

$$\|X_r - X_s\|^2 = (X_r - X_s)^T(X_r - X_s)$$

But when the variation in X is stochastic in nature it is better to look at a transformation of the form

$$Z_r = S^{-\frac{1}{2}}\left(X_r - \overline{X}\right).r = 1,2,3, \dots n$$

This enables one to eliminate the correlation between the variables and standardize the variances of each variable.

$$S = \frac{1}{n}\sum_{r=1}^{n}(X_r - \overline{X})\left(X_r - \overline{X}\right)^T$$

After the aforesaid transformation one can look at the Euclidean distance between the transformed rows. Such distances play a role in cluster analysis. The most important of these distances is the Mahalanobis distance given by

$$D_{rs}^2 = \|Z_r - Z_s\|^2 = (X_r - X_s)^T S^{-1}(X_r - X_s)$$

Mahalanobis distance can be of different kinds.

- Let X ~ $\mu_1, \sum$ and let Y ~ $(\mu_2, \sum)$) then $D_{\mu_1\mu_2}$ is a Mahalanobis distance between the parameters.
- Let X ~ $(\mu, \sum)$. The Mahalanobis distance betweenX andμ, $D_{X,\mu}$, is here a random variable.
- Let X ~ $(\mu_1, \sum)$, Y ~ $(\mu_2, \sum)$. The Mahalanobis distance between X and Y is $D_{X,Y}$ [2,3].

Merits of the D^2 control chart

D^2 control chart exercises simultaneous control on proportion defectives falling in various categories of defects in a single chart without any distortion in the advertised level of type I error.

While constructing a *p*-chart it is a standard practice to draw *p*-charts for major defects only along with the overall *p*-chart. Hence, the conventional *p*-charts do not take care of the fluctuations involved in the categories of minor defects – the cumulative effect of which may very well destabilize the process at times. However, the D^2 control chart takes into account various categories of defects exhaustively and thereby enhances the sensitivity in the detection of a shift. Therefore, to understand the overall performance of a process by considering all categories of defects at the same time, the D^2 control chart is very effective.

Example

This case example is concerned with the proportion defective data with regard to various kinds of paint defects of a ceiling fan cover. The defects, which are prevalent in painting of such covers, are poor covering, overflow, patty defect, bubbles, paint defects, buffing defects. Apart from the six categories of defects the good items are also considered for computing the D^2 statistic. Hence, the value of *K* in this particular case is 7 and the value of *n* or the number of objects or time periods is 24. For any time period $\sum_{j=1}^{7} p_{ij} = 1$, *i* = 1, 2, 3, . . . , 24. Here p_{ij} is the proportion defective or proportion good item at a particular time period *i* for each *j*. It can be seen from Table 1 that the sample size ranges from 20 to 404. The D^2 chart and the corresponding overall *p*-chart and *p*-charts for the individual defect categories are given in Figures 1 to 8 for ready comprehension. Note that this is a perfect multinomial case in the sense that as soon as a particular cover is found to contain a defect it is categorized into the most predominant defect it has within. Hence, a particular defective cover contains one and only one kind of defect.

Table 1. Data for paint defect.

Time period	No. inspected	Poor covering	Overflow	Patty defect	Bubbles	Paint defect	Buffing	Total
1	176	15	12	2	6	3	4	42
2	160	13	18	4	4	5	5	49
3	186	18	12	6	8	4	6	54
4	167	12	10	5	4	3	2	36
5	291	8	6	4	3	2	5	28
6	170	10	6	2	3	2	3	26
7	224	15	12	4	3	6	5	45
8	140	11	9	5	4	3	4	36
9	250	19	15	7	4	8	5	58
10	145	20	10	4	2	6	2	44
11	100	15	12	2	3	3	4	39
12	80	4	2	1	2	2	1	12
13	170	13	11	5	1	4	3	37
14	200	22	17	4	3	6	2	54
15	112	8	6	2	3	1	2	22
16	250	15	11	4	5	7	0	42
17	122	12	9	6	8	4	5	44
18	312	23	18	4	7	5	9	66
19	200	15	13	4	6	7	5	50
20	20	3	2	2	1	0	0	8
21	60	6	4	2	2	3	2	19
22	404	40	15	6	8	4	20	93
23	104	8	6	2	3	2	4	25
24	124	12	9	3	4	2	5	35
Total	4167	337	245	90	97	92	103	964

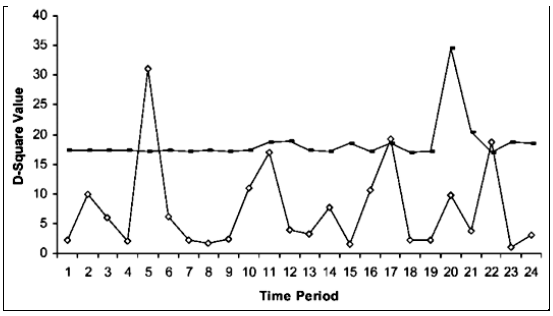


Figure 1. D^2 chart

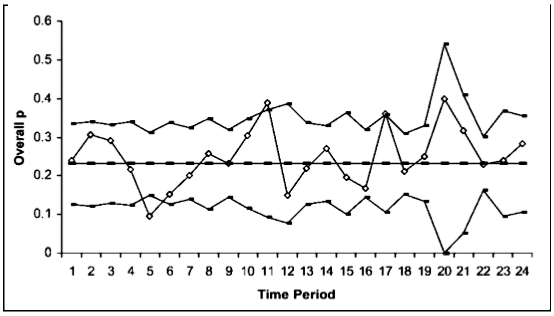


Figure 2. Overall *p*-chart

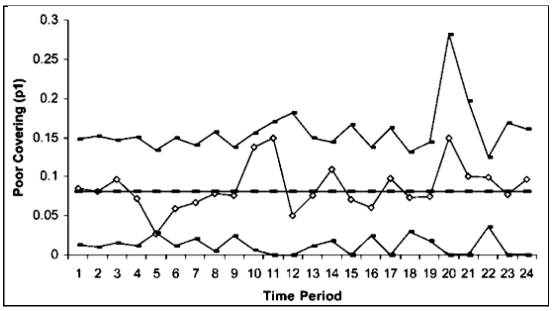


Figure 3. *p*-chart for poor covering

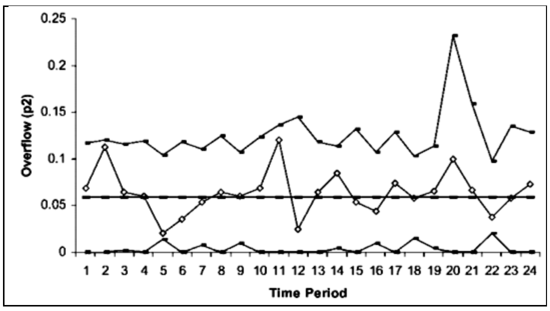


Figure 4. *p*-chart for overflow

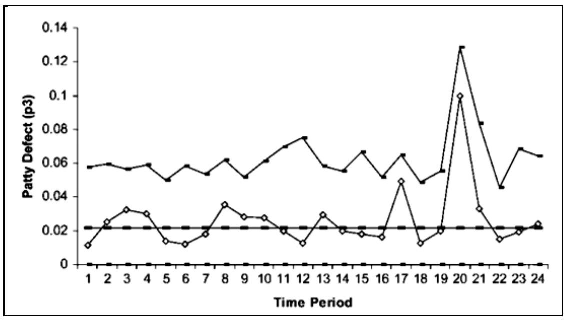


Figure 5. *p*-chart for patty defect

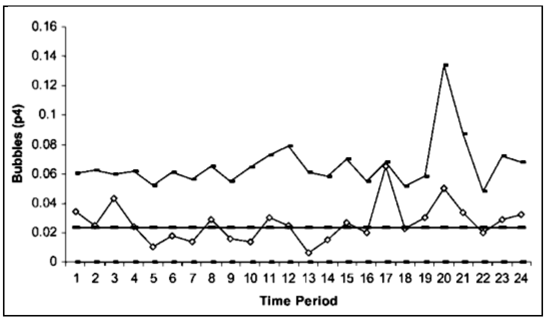


Figure 6. *p*-chart for bubbles

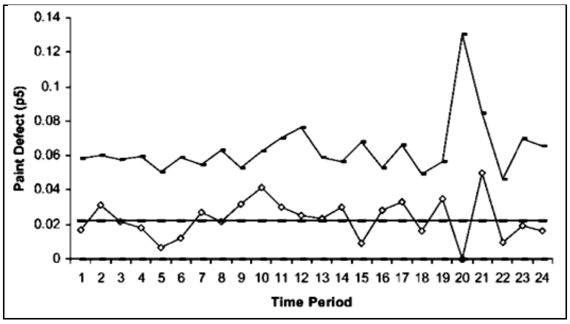


Figure 7. ***p*****-chart for paint defect**

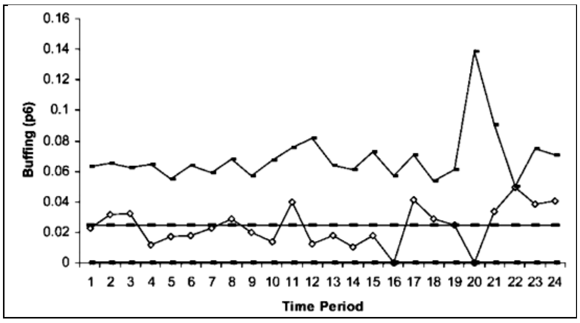


Figure 8. ***p*****-chart for buffing**

Observation and interpretation

It may be observed from the D^2 chart that the points 5, 17, and 22 have fallen outside the control limit (1%). In addition, it may also be noted that point number 11 (D^2 value 16.9897) has fallen outside the warning limit (5%), the value of which is 13.7053. Note that if α is the type I error for the D^2 chart and α is the type I error for the individual p-charts then $\alpha = 1 - (1 - \alpha)K$. For $\alpha = 0.01$, α is found to be 0.001 for $K = 7$ categories. So for individual p-charts 3.19σ control limits have been used. At the 5th time period the overall proportion defective and the proportion defective due to poor covering lie below their respective lower control limit. Other proportion defectives do not show any lack of statistical control but their simultaneous low values have got a conspicuous impact on the D^2 chart. At the 17th time period the overall proportion defective has fallen outside the upper control limit and the ‘patty defect’ and ‘bubbles’ have fallen on the higher side nearer to their respective upper control limits. The D^2 chart has revealed that. At the 22nd time period the defect named as ‘buffing’ shows an out of control situation since the corresponding proportion defective falls beyond the upper.