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Course Title: Statistical Quality Control and Reliability Theory

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Unit-I

Statistical Quality Control

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STATISTICAL QUALITY CONTROL AND RELIABILITY – 23ST08CC

UNIT - I

Definition of Quality

The meaning of quality is closely allied to cost and customer needs quality may simply be defined is fitness for purpose at lowest cost.

The component is said to posses' good quality if it works well in the equipment for which it is meant quality is thus defined as "fitness for purpose".

1. Quality of Design
2. Quality of Conformance

Quality is inversely proportional to variability.

Basic concepts of quality

Every manufacturing organisation is concerned with the quality of its product while it is important that quality requirement be satisfied and production schedules meet it is quality equally important that the finished product meet established specification. Because, customer satisfaction is derived from quality products and service.

Meaning of Control

Control is a system for measuring and checking (inspecting) a phenomenon. It suggests what to inspect how often to inspect and how much to inspect. In addition, it incorporates a feedback mechanism which explores the causes of poor quality and takes corrective action.

Control differs from inspection as it ascertains quality characteristics of an item, compares the same with prescribed quality standards and separates defective terms from non-defective ones-inspection. However does not involve any mechanism to take corrective action.

Meaning of quality control

- Quality control is a systematic control of various factors that affect the quality of the product. The various factors include material, tools machines, types of labour, working conditions, measuring instruments, etc,.
- Quality control can be defined as the entire collection of activities which ensures that the operation will produce the optimum quality products at minimum cost.
- As per A.Y. Feigorbarum total quality control is "An effective system for integrating the quality development". Quality maintenances and quality improvement efforts of the various groups in an organisation. So as to enable production and services at the most economical level which allow full customer satisfaction.

- In the words of Alford and Beatly “quality control” may be broadly defined as that “Industrial management technique means of which products of uniform accepted quality are manufactured”. Quality control is concerned with making things right rather than discovering and rejecting those made wrong.
- In short, we can say that quality control is a technique of management for achieving required standard of products.

Classification of quality control

Quality control therefore covers all the factors and processes of production which may be broadly classified as follows;

- **Quality of materials**

  Material of good quality will result in smooth processing thereby reducing the waste and increasing the output. It will also give better finish to the end products.
- **Quality of manpower**

  Trained and qualified personnel will give increased efficiency due to the batter quality production through the application of skill and also reduce production cost and waste
- **Quality of machines**

  Better quality equipment will result in efficient work due to lack or scarcity of breakdowns and thus reduce the cost of defectives.
- **Quality management**

  A good management is imperative for increase in efficiency, harmony in relatives, and growth of business and markets.

Basis of statistical quality control

The basis of statistical quality control is the degrees of variability to the size or the magnitude of a given characteristic of the product. Variation in the quality of manufactured product in the repetitive process in industry is inherent and inevitable. These variations are broadly classified as being due to two causes, viz.,

i) Chance causes and
ii) Assignable causes.

Chance causes

- ‘Stable pattern of variation’ or ‘a constant causes system’ is inherent in any particular scheme of production and inspection. This pattern results from many minor causes that behave in a random manner. The variation due to these causes is beyond the control of human hand and cannot be prevented or eliminated under any circumstances.
- One has got to allow for variation within this stable pattern, usually termed as allowable variation. The range of such variation is known as ‘natural tolerance of the processes.

Assignable causes

- The second type of variation attributed to any production process is due to non random or the so called the Assignable causes and is termed as preventable variation. The Assignable causes may creep in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods.
- Some of the important factors of assignable causes of variation are sub standard or defective raw materials, new techniques or operations, negligence of the operators, wrong or improper handling of machines, faculty equipment, unskilled or inexperienced technical staff and so on.
- These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong, i.e., before the production becomes defective.

Chance causes of variation	Assignable causes of variation
i) Consist of many individual causes. ii) Any one chance cause results in only a small amount of variation iii) Chance variation cannot economically be estimated from a process. iv) Some typical chance causes of variation are: • Slight vibration of a machine. • Lack of human perfection in reading instruments and setting controls. • Voltage fluctuations and variation in temperatures.	Consist of just a few individual causes. Any one assignable cause can result in a large amount of variation. The presence of assignable variation can be detected, and action to eliminate the causes is usually economically justified. Some typical assignable causes of variation are: • Negligence of operators • Defective raw materials • Faulty equipment • Improper handling of machines

Definition of Statistical Quality Control

"S.Q.C. may be broadly defined as that industrial management technique by means of which product of uniform acceptable quality is manufactured. It is mainly concerned with setting things right rather than discovering and rejecting those made wrong." - Duncan

Benefits of Statistical Quality Control

The following are some of the benefits that result when a manufacturing process is operating in a state of statistical control:

✓ An obvious advantage of S.Q.C. is the control, maintenance and improvement in the quality standards.

✓ The act of getting a process in statistical quality control involves the identification and elimination of assignable causes of variation and possibly the inclusion of good ones, viz., new material or methods. This (i) helps in the detection and correction of many production troubles, and (ii) brings about a substantial improvement in the product quality and reduction of spoilage and rework.

- ✓ It tells us when to leave a process alone and when to take action to correct troubles, thus preventing frequent and unwarranted adjustments.
- ✓ If a process in control (which is doing about all we can expect of it) is not good enough, we shall have to make more or less a radical (fundamental) change in the process-just meddling (tampering) with it won't help.
- ✓ A process in control is predictable-we know what it is going to do and thus we can more safely guarantee the product. In the presence of good statistical control by the supplier, the previous lots supply evidence on the present lots, which is not usually the case if the process is not in control?
- ✓ It testing is destructive (e.g., testing the breaking strength of chalk ; proofing of ammunition, explosives. Crackers, etc.), a process in control gives confidence in the quality of untested product which is not the case otherwise.
- ✓ It provides better quality assurance at lower inspection cost.
- ✓ Quality control finds its applications not only in the sphere of production, but also in other areas like packaging, scrap and spoilage, recoveries, advertising, etc. Foreign trade items of developing countries like India are particularly appropriate for every type of quality control in every possible area.
- ✓ The very presence of a quality control scheme in a plant improves and alerts the personnel. Such a scheme is likely to breed 'quality consciousness' throughout the organisation which is of immense long-run value.
- ✓ S.Q.C. reduces waste of time and material to the absolute minimum by giving an early warning about the occurrence of defects. Savings in terms of the factors stated above mean less cost of production and hence may ultimately lead to more profits.

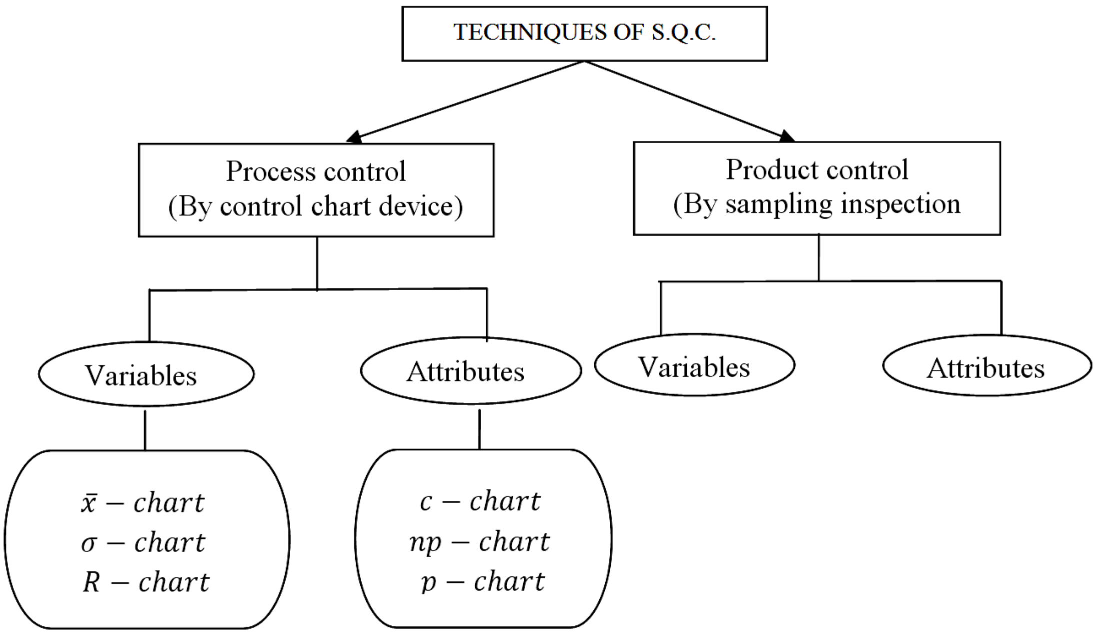
Process control

The main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product so that it conforms to specified quality standards.

In other words, we want to ensure that the proportion of defective items in the manufactured product is not too large. This is termed as 'Process control' and is achieved through the technique of 'control charts' pioneered by W.A.Shewhart in 1924.

Product control

Product control we mean controlling the quality of the product by critical examination at strategic points and this is achieved through 'Sampling Inspection Plans' pioneered by H.F.Dodge and H.C.Romig.



Techniques of SQC

Control limits:

These limits of sampling variation of a statistical measures (e.g., mean, range or fraction defective) such that if the production process is under control the values of the measures calculated item different rational sub group will lie within these limits.

Points falling outsides the control limits indicates that the process to not operating under a system of chance causes i.e., assignable causes of variation are present which must be eliminated control limits are used in control charts.

Specification limits:

The manufactured have to be decide upon the maximum and the minimum allowable dimensions of quality chart so that the product can be gainfully utilized for which it to intended. If the dimensions are beyond these limits the product to treated a defective and cannot be used. These maximum and minimum limits of variations of individual's items as maintained in the product design are known as specification limits.

Tolerance limits:

These are limits of variations of quality measures of the product between which at least a specified proportion of the product is expected to we provided the process to in a state os statistical quality control.

Control charts

Shewhart's control charts provide a powerful tool of discovering and correcting the assignable causes of variation outside the 'stable pattern' of chance causes, thus enabling us to stabilize and control our processes at desired performances and thus bring the process under statistical control. In industry one is faced with two kinds of problems:

i) To check whether the process is conforming to standards laid down, and
ii) To improve the level of standard and reduce variability consistent with cost considerations.

Shewhart's control charts provide an answer to both. A typical control chart consists of the following three horizontal lines:

i) A central line (C.L.), indicating the desired standard or the level of the process.
ii) Upper control limit (U.C.L.), indicating the upper limit of tolerance.
iii) Lower control limit (L.C.L.), indicating the lower limit of tolerance.

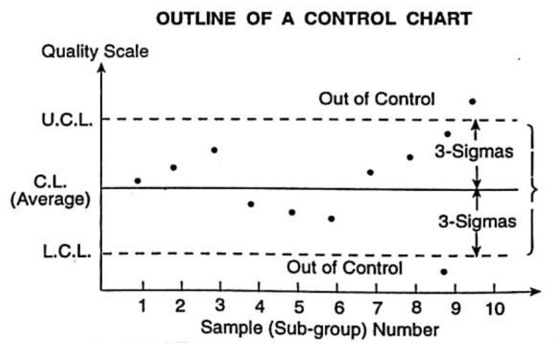
The control line as well as the upper and lower limits are established by computations based computations based on the past records or current production records.

Major parts of a control chart:

A control chart generally includes the following four major parts:

- **Quality Scale :** This is a vertical scale. The scale is marked according to the quality characteristics (either in variables or in attributes) of each samples.
- **Plotted samples :** The qualities of individual items of a sample are not shown on a control chart. Only the quality of the entire sample represented by a single value (a statistic) is plotted. The single value plotted on the chart is in the form of a dot (sometimes a small circle or a cross).
- **Sample (or sub-group) Numbers :** The samples plotted on a control chart are numbered individually and consecutively on a horizontal line. The line is usually placed at the bottom of the chart. The samples are also referred to as sub-groups in statistical quality control. Generally 25 sun-groups are used in constructing a control chart.
- **Horizontal lines :** The central line represents the average quality of the samples plotted on the chart. The line above the central line shows the upper control limit (U.C.L.) which is commonly obtained by adding 3 sigma's to the average, i.e., mean+3(S.D.) the line below the central line is the lower control limit (L.C.L.) which is obtained by subtracting 3 sigma's from the average, i.e., mean-3(S.D.) the upper and lower control limits are usually drawn as dotted lines, and the central line is plotted as a bold (dark) line.

The adjoining diagram depicts the principles of Shewhart's control chart.



In the control chart, upper control limit (U.C.L.) and lower control limit (L.C.L.) are usually plotted as a bold (dark) line. If t is the underlying statistic then these values depend on the sampling distribution of t and are given by:

$$UCL = E(t) + 3S.E(t)$$
$$LCL = E(t) - 3S.E(t)$$
$$CL = E(t)$$

Tools for S.Q.C

The following four, separate but related techniques, are the most important statistical tools for data analysis in quality control of the manufactured products:

- **Shewhart's control chart for variables**.

  i.e., for a characteristic which can be measured quantitatively. Many quality characteristic of a product are measurable and can be expressed in specific units of measurements such as diameter of a screw, tensile strength of steel pipe, specific resistance of a wire, life of an electric bulb, etc. Such variables are of continuous type and are regarded to follow normal probability law. For quality control of such data, two types of control charts are used and technically these charts are known as:

  (a) Charts for $\bar{x}$ (mean) and R (range), and

  (b) Charts for $\bar{x}$ (mean) and σ (standard deviation)

- **Shewhart's control chart for fraction defective or p-chart.**

  This chart is used if we are dealing with attributes in which case the quality characteristics of the product are not amenable to measurement but can be identified by their absence or presence from the product or by classifying the product as defective or non-defective.

- **Shewhart's control charts for the Number of defects per unit or c-chart.**

  This is usually used with advantage when the characteristic representing the quality of a product is a discrete variable, e.g.,

  a. the number of defective rivets in an aircraft wing, and
  b. the number of surface defects observed in a roll of coated paper or a sheet of photographic film.

- The portion of the sampling theory which deals with the quality protection given by any specified sampling acceptance procedure.

Advantages of control charts

- ✓ A control charts indicates whether the process is in control or out of control.
- ✓ It determines process variability and detects unusual variations taking place in a process.
- ✓ It ensures product quality level.
- ✓ It warns in time, and if the process is rectified at that time, scrap or percentage rejection can be reduced.
- ✓ It provides information about the selection of process and setting of tolerance limits.
- ✓ Control charts build up the reputation of the organization through customer's satisfaction.

Disadvantages of control charts

- ✓ These charts depend on accurate and reliable data. If the data collection is flawed or incomplete, it can result in inaccurate interpretations and erroneous decision-making.
- ✓ They require advanced statistical knowledge and expertise for accurate data interpretation. Misinterpretation of chart patterns or inaccurate analysis can lead to ineffective measures and missed opportunities for improvement.
- ✓ Implementing these charts requires time, effort, and resources to gather and analyze the data, set control limits, and maintain the tracking process. Thus, employing these charts can be challenging for organizations with limited resources.

Control charts for variables:

These charts may be applied to any quality characteristic that is measurable. In order to control a measurable characteristic we have to exercise control on the measure of location as well as the measure of dispersion. Usually $\bar{X}$ and R charts are employed to control the mean (location) and standard deviation (dispersion) respectively of the characteristic.

- The mean chart (or) $\bar{\boldsymbol{X}}$ - chart
- The range chart (or) R – chart
- σ - Chart

$\bar{\boldsymbol{X}}$ and R charts

No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristics of the produced process viz., raw material, machine setting and handling, operators, etc. As pointed out earlier, this variation is the result of

i) Chance causes
ii) Assignable causes

The control limits in the $\bar{x}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the

- ✓ Average – mostly related to machine setting and
- ✓ Range – mostly related to negligence on the part of the operator.

Control limits for $\overline{X}$– chart

Upper control limit U.C.L. = $\mu + \frac{3\sigma}{\sqrt{n}}$

Lower control limit L.C.L. = $\mu - \frac{3\sigma}{\sqrt{n}}$

μ is estimated as $\bar{\bar{X}} = \frac{\sum x_i}{N}$,

where,

- N is the number of samples.
- σ is estimated $\frac{\bar{R}}{d_2}$

where,

$\bar{R} = \frac{\sum R_i}{N}$ and d_2 is constant.

Derived the basis of n, Hence the control limits are redefined

$$\text{UCL} = \bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}} \text{ and LCL} = \bar{\bar{X}} - \frac{3\bar{R}}{d_2\sqrt{n}}$$

Take $A_2 = \frac{3}{d_2\sqrt{n}}$, the control limits are obtained as

$$\text{UCL} = \bar{\bar{X}} + A_2\overline{R} \text{ and LCL} = \bar{\bar{X}} - A_2\overline{R}.$$

Control limits for R- chart

$$\text{UCL} = D_4\overline{R} \text{ and LCL} = D_3\overline{R}$$

where D_3 *and* D_4 are constant values depending on the sample size (n). The values of D_3 *and* D_4 are given in the table quality control chart constants.

Interpretation of $\overline{X}$ and R charts

Sl. No.	Situation in		Interpretation
	R chart	**$\overline{X}$ chart**	
1	In control	Points beyond limits only on one side.	Level of process has shifted
2	In control	Points beyond limits on both the sides.	Level of process is changing in erratic manner frequent adjustments
3	Out of control	Points beyond limits on both the sides.	Variability has increased.
4	Out of control	Out of control on one side	Both level and variability have changed.
5	In control	Run of 7 or more points on one side of control line	Shifted in process level.
6	In control	Trend of 7 or more points. No point outside control limits	Process level is gradually changing.
7	Runs of 7 or more points above central line	-	Variability has increased.
8	Points too close to the central line	-	Systematic differences within sub groups
9	-	Points too close to the central line	Systematic differences within sub groups

Problems for $\overline{X}$ chart and R chart:

EXAMPLE 1:

Aim: Construct a control chart for mean and range for the following data on the basis of fuses, samples of 4 being taken every hour.

S. No.	Observations				S. No.	Observations			
	1	2	3	4		1	2	3	4
1	27	23	36	24	14	28	30	22	41
2	30	18	27	32	15	44	32	22	41
3	21	44	22	28	16	26	42	35	28
4	40	21	29	24	17	38	40	51	32
5	51	34	17	10	18	26	28	34	39
6	33	30	28	22	19	42	38	52	36
7	30	22	18	12	20	30	32	39	45
8	20	34	15	42	21	23	44	48	33
9	35	48	20	47	22	28	34	39	44
10	22	50	45	41	23	25	29	44	33
11	34	22	36	44	24	30	30	40	32
12	32	48	32	33	25	38	27	39	22
13	34	32	28	38					

Comment on whether the production seems to be under control. Assuming that these data are the first one.

Procedure:

- Compute $\overline{\mathbf{X}}$ - chart and R chart for each samples using the formula

$$\bar{\bar{X}} = \frac{\sum x_i}{k}, \text{ where, k is the number of observations.}$$

$$\mathbf{R_i = max(X_i) - min(X_i)}.$$

- Calculate $\bar{\bar{X}}$ and $\bar{R}$ for overall data using the formula,

$$\bar{\bar{\mathbf{X}}} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \frac{\bar{\mathbf{X}}_\mathbf{i}}{\mathbf{n}}, \bar{\boldsymbol{R}} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \frac{\mathbf{R_i}}{\mathbf{n}}$$

  where n = number of sample

- Calculate the control limits for $\bar{X}$-chart using the formula,

  Upper control limit $\mathrm{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ and

  Lower control limit $\mathrm{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$.

  Center line $\mathrm{CL}_{\bar{X}} = \bar{\bar{X}}$

- Calculate the control limits for R-chart using the formula

  Upper control limit $\mathrm{UCL_R} = D_4\bar{R}$

  Lower control limit $\mathrm{LCL_R} = D_3\bar{R}$

  Center line $\mathrm{CL_R} = \bar{\bar{R}}$

  Where A_2, D_3, D_4 are constants from control charts table

- Plot the limits and Samples in the chart and interpret the result**.**

Calculation:

S. No.	Observations				$\bar{X}$	R
	1	2	3	4		
1	27	23	36	24	27.5	13
2	30	18	27	32	26.75	14
3	21	44	22	28	28.75	23
4	40	21	29	24	28.5	19
5	51	34	17	10	28	41
6	33	30	28	22	28.25	11
7	30	22	18	12	20.5	18
8	20	34	15	42	27.75	27
9	35	48	20	47	37.5	28
10	22	50	45	41	39.5	28
11	34	22	36	44	34	22
12	32	48	32	33	36.25	16
13	34	32	28	38	33	10
14	28	30	22	41	30.25	19
15	44	32	22	41	34.75	22
16	26	42	35	28	32.75	16
17	38	40	51	32	40.25	19
18	26	28	34	39	31.75	13
19	42	38	52	36	42	16
20	30	32	39	45	36.5	15
21	23	44	48	33	37	25
22	28	34	39	44	36.25	16
23	25	29	44	33	32.75	19
24	30	30	40	32	35	10
25	38	27	39	22	31.5	17

From the table given in the end of the chapter, n = 25, A_2= 0.729, D_3 = 0, D_4 = 2.282

$$\sum_{i=1}^{n} \bar{X}_i = 817 \text{ and } \sum_{i=1}^{n} R_i = 477$$

$$\bar{\bar{\mathbf{X}}} = \sum_{i=1}^{n} \frac{\bar{\mathbf{X}}_i}{\mathbf{n}} = \frac{\mathbf{817}}{\mathbf{25}} = \mathbf{32.68}$$

$$\bar{\boldsymbol{R}} = \sum_{i=1}^{n} \frac{\mathbf{R_i}}{\mathbf{n}} = \frac{\mathbf{477}}{\mathbf{25}} = \mathbf{19.08}$$

$\bar{\bar{X}}$- Chart

Upper control limit $UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ =32.68 + (0.729) (19.08) $= 46.58$

Lower control limit $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$ =32.68 – (0.729) (19.08) =18.77

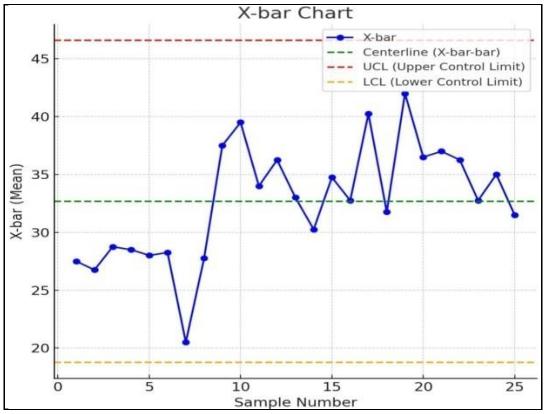
Center line $CL_{\bar{X}} = 32.68$

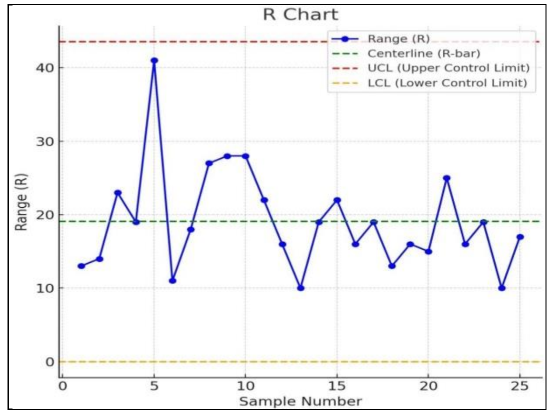
R-chart

Upper control limit $UCL_R = D_4\bar{R}$ = **(2.282) (19.08)** $= 43.54$

Lower control limit $LCL_R = D_3\bar{R}$ = (0) (19.08) $= 0$

Center line $CL_R = \bar{\bar{R}} = 19.08$





Result:

In $\overline{\overline{\mathbf{X}}}$-chart, we see that all the points lie inside the control limit; the process variability is also in control.

In R-chart, we see that all the points lie inside the control limit, the process variability is also in control

EXAMPLE 2:

Aim: The following $\overline{\mathrm{X}}$ and R values of 20 subgroups of 5 readings. Determine the control limits for $\overline{\mathrm{X}}$ and R-charts for future use, eliminating all the out of controls.

Subgroups	Mean	Range	Subgroups	Mean	Range
1	34	4	**11**	35.8	4
2	31.8	4	**12**	38.4	4
3	30.6	2	**13**	34	14
4	33	3	**14**	35	4
5	35	5	**15**	33.8	7
6	32.2	2	**16**	31.6	5
7	33.6	5	**17**	33	5
8	32	13	**18**	28.2	3
9	33.8	19	**19**	31.8	9
10	37.8	6	**20**	35.6	6

Procedure:

- Compute $\overline{\boldsymbol{X}}$ - chart and R chart for each samples using the formula $\overline{\overline{\boldsymbol{X}}} = \frac{\sum x_i}{k}$, where, k is the number of observations.

$$\mathbf{R_i} = \mathbf{max}(\mathbf{X_i}) - \mathbf{min}(\mathbf{X_i}).$$

- Calculate $\overline{\overline{\mathrm{X}}}$ and $\overline{\mathrm{R}}$ for overall data using the formula,

$$\overline{\overline{\mathrm{X}}} = \sum_{i=1}^{n} \frac{\overline{\mathrm{X}}_i}{n}, \bar{R} = \sum_{i=1}^{n} \frac{R_i}{n}$$

where n = number of sample

- Calculate the control limits for $\bar{X}$-chart using the formula,
  Upper control limit $UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ and
  Lower control limit $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$.
  Center line $CL_{\bar{X}} = \bar{\bar{X}}$
- Calculate the control limits for R-chart using the formula
  Upper control limit $UCL_R = D_4\bar{R}$
  Lower control limit $LCL_R = D_3\bar{R}$
  Center line $CL_R = \bar{R}$
  Where A_2, D_3, D_4 are constants from control charts table
- Plot the limits and Samples in the chart and interpret the result.

Calculation:

From the table given in the end of the chapter,

$$n = 20,\ \boldsymbol{A_2} = 0.577,\ \boldsymbol{D_3} = 0,\ \boldsymbol{D_4} = 2.114$$

$$\sum_{i=1}^{n} \bar{X}_i = 671, \sum_{i=1}^{n} R_i = 124$$

$$\bar{\bar{\mathbf{X}}} = \sum_{i=1}^{n} \frac{\bar{\mathbf{X}}_i}{\mathbf{n}} = \frac{\mathbf{671}}{\mathbf{20}} = \mathbf{33.55}$$

$$\bar{\boldsymbol{R}} = \sum_{i=1}^{n} \frac{\mathbf{R_i}}{\mathbf{n}} = \frac{\mathbf{124}}{\mathbf{20}} = \mathbf{6.2}$$

$\bar{\bar{X}} - C$hart

Upper control limit $UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$ =33.55 + (0.577) (6.2) = 37.12

Lower control limit $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$ =33.55– (0.577) (6.2) = 29.97

Center line $CL_{\bar{X}} = 33.55$

R-chart

Upper control limit $UCL_R = D_4\bar{R}$ **= (2.114) (6.2)** = 13.10

Lower control limit $LCL_R = D_3\bar{R}$ = (0) (6.2) = 0
Center line $CL_R = \bar{\bar{R}} = 6.2$

On plotting the control charts it will be noticed that the sample points (mean) corresponding to the sub-groups 10 and 12 fall above $UCL_{\bar{X}}$ and sub-group 18 falls below $LCL_{\bar{X}}$.

Hence, the process is out of control, since out control points suggest the presence of assignable of variation, eliminating these points we have,

$$\bar{X}_{Revised} = \frac{671 - (37.8 + 38.4 + 28.2)}{20 - 3} = 33.33$$

On plotting the R-chart, it will be noticed that the sample ranges for sub-groups 9 and 13 fall above $\mathbf{UCL_R}$ **and** the process is out of Control. Eliminating these points, we have,

$$\bar{R}_{Revised} = \frac{124 - (19 + 14)}{20 - 3} = 5.06$$

Revised control limit for the future:

$\mathbf{\bar{X}_{Revised}}$ **– chart**:

Upper control limit $\mathrm{UCL}_{\bar{X}_{\mathrm{revised}}} = \bar{\bar{X}}_{revised} + A_2\bar{R}_{revised}$ =33.33 + (0.577)(5.05) = 36.24

Lower control limit $\mathrm{LCL}_{\bar{X}_{\mathrm{revised}}} = \bar{\bar{X}}_{revised} - A_2\bar{R}_{revised}$ =33.55– (0.577) (5.05) = 30.41

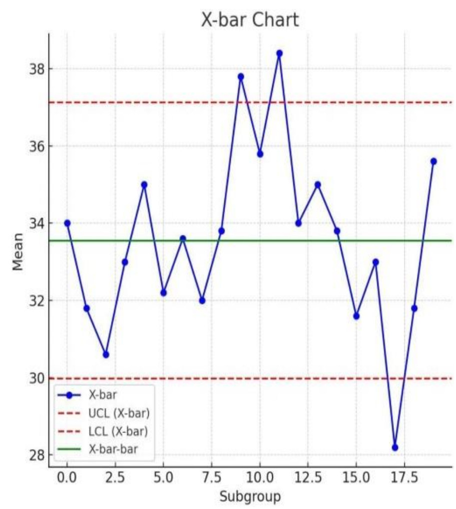
Center line $\mathrm{CL}_{\bar{X}_{\mathrm{revised}}} = 33.33$

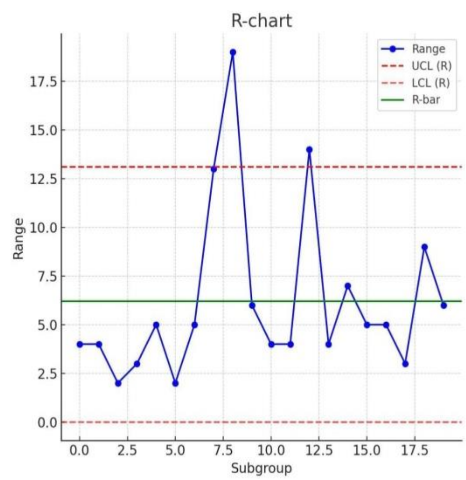
$\mathbf{R_{revised}}$ **-chart**

Upper control limit $\mathrm{UCL}_{\mathrm{R}_{\mathrm{revised}}} = D_4\bar{R}_{revised}$ = (2.114) (5.06) = 10.69

Lower control limit $\mathrm{LCL}_{\mathrm{R}_{\mathrm{revised}}} = D_3\bar{R}_{revised}$ = (0) (5.06) = 0

Center line $\mathrm{CL}_{\mathrm{R}_{\mathrm{revised}}} = \bar{R}_{revised} = 5.06$





Result:

- In $\overline{\mathrm{X}}$-chart, none of the remaining 17 sample points lie outside revised control limits $\left[\mathbf{UCL}_{\mathbf{X}_{\text{revised}}} = \mathbf{36.24}, \mathbf{UCL}_{\mathbf{X}_{\text{revised}}} = \mathbf{30.41}\right]$, these may be regarded as the control limits $\overline{\mathrm{X}}$-chart for the future production from this process.
- In R-chart, none of the remaining 18 sample points lie outside revised control limits $\left[\mathbf{UCL}_{\mathbf{R}_{\text{revised}}} = \mathbf{10.69},, \mathbf{UCL}_{\mathbf{R}_{\text{revised}}} = \mathbf{0.}\right]$, these may be regarded as the control limits

  R-chart for the future production from this process.

Control charts for Standard Deviation (or σ – Chart)

Since Standard Deviation is an ideal measure of dispersion, a combination of Control chart for mean ($\bar{x}$) and Standard Deviation(s), known as $\bar{X}$ and s charts (or $\bar{x}$ and σ charts) is theoretically more appropriate then a combination of $\bar{X}$ and R charts for controlling process average and process variability. In a random sample of size n from normal population with Standard Deviation σ, we have

$$E(s^2) = \frac{n-1}{n}\sigma^2 \text{ and } E(s) = C_2\sigma, \text{ where } C_2 = \sqrt{(2/\pi)}\ \frac{[(n-2)/2]!}{[(n-3)/2]!}$$

Hence, in sampling from normal population, we have

$$var(s) = E(s^2) - [E(s)]^2 = \left(\frac{n-1}{n} - {C_2}^2\right)\sigma^2$$

$$\text{S.E. (s)} = C_3.\sigma, where\ C_3 = \sqrt{\frac{n-1}{n} - {C_2}^2}$$

$$\mathrm{UCL_s} = E(s) + 3.\mathrm{S.E.(s)} = (C_2 + 3C_3)\sigma = B_2.\sigma$$

$$\mathrm{LCL_s} = E(s) - 3.\mathrm{S.E.(s)} = (C_2 - 3C_3)\sigma = B_1.\sigma$$

$$\text{Central line} = CL_s = C_2.\sigma$$

If the value of σ is not specified or not known, then we use its estimate, based on $\bar{s}$ given by

$$\hat{\sigma} = \frac{\bar{s}}{C_2}.$$

$$\mathrm{UCL_s} = E(s) + 3.\mathrm{S.E.(s)} = \bar{s} + 3\frac{C_3}{C_2}\bar{s} = \left(1 + 3\frac{C_3}{C_2}\right)\bar{s} = B_4\bar{s}$$

Similarly, we shall get

$$\mathrm{LCL_s} = E(s) - 3.\mathrm{S.E.(s)} = \bar{s} - 3\frac{C_3}{C_2}\bar{s} = \left(1 - 3\frac{C_3}{C_2}\right)\bar{s} = B_3.\bar{s} \text{ and}$$

$$CL_s = \bar{s}$$

where $B_3\ and\ B_4$ have been tabulated for different values of n.

Since s can never be negative, if LCL comes out to be negative, as will be the case for n from 2 to 5, it is taken to be zero.

Control charts for Attributes

In spite of wide applications of $\bar{X}$ and R (or σ) charts as a powerful tool of diagnosis of sources of trouble in a production process, their use is restricted because of the following limitations;

- They are charts for variables only, i.e., for quality characteristics which can be measured and expressed in numbers.
- In certain situations they are impracticable and un economical, e.g., if the number of measurable characteristics, each of which could be a possible candidate for $\bar{X}$ and R charts, is too large, say 30000 or so then obviously there cannot be 30000 control charts.

As an alternative to $\bar{X}$ and R charts, we have the control charts for attributes which can be used for quality characteristics:

- ✓ Which can be observed only as attributes by classifying an item as defective or non defective i.e., conforming to specifications or not, and
- ✓ Which are actually observed as attributes even though they could be measured as variables i.e., go and no-go gauge test results.

They are control charts for attributes:

- Control chart for fraction defective (p- Chart)
- Control chart for the number of defectives (np- Chart or d chart)
- Control chart for the number of defects per unit (c-Chart)

(i) Control chart for fraction defective (p- Chart)

- While dealing with attributes, a process will be adjudged in statistical control of all the samples or subgroups are ascertained to have the same population proportion P.
- If 'd' is the number of defectives in a sample of size n, then the sample proportion defective is $p = \frac{d}{n}$. hence, d is a binomial variate with parameters n and P.

$$E(d) = nP \quad and \quad var(d) = nPQ, \qquad Q = 1 - P$$

Thus

$$E(p) = E\left(\frac{d}{n}\right) = \frac{1}{\mathrm{n}}\mathrm{E(d)} = \mathrm{P} \;\; and \;\; var(p) = var\left(\frac{d}{n}\right) = \frac{1}{n^2}\mathrm{var\ (d)} = \frac{PQ}{n} \quad \rightarrow (1)$$

Thus, the 3-$\boldsymbol{\sigma}$ control limits for p- chart are given by**:**

$$E(p) \pm 3S.E(p) = P \pm 3\sqrt{PQ/n} = P \pm A\sqrt{PQ}$$

where $A = 3/\sqrt{n}$ has been tabulated for different values of n.

Case (I) Standards Specified.

If p' is the given or known value of P, then

$$\mathrm{UCL_p} = \mathrm{p}' + \mathrm{A}\sqrt{P'(1-P')}; \quad \mathrm{LCL_p} = \mathrm{p}' - \mathrm{A}\sqrt{P'(1-P')}; \;\; \mathrm{CL_p} = \mathrm{p}'$$

Case (II) Standards Not Specified.

Let d_i be the number of defectives are p_i the fraction defective for the ith sample (i=1, 2, ..., k) of size n_i then the population proportion P is estimated by the statistic $\bar{p}$ given by;

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

It may be remarked here the $\bar{p}$ is an unbiased estimate of P, since

$$E(\bar{p}) = \frac{\sum E(d_i)}{\sum n_i} = \frac{\sum n_i P}{\sum n_i} = P$$

In this case

$$\mathrm{UCL_p} = (\bar{p}) + \mathrm{A}\sqrt{\bar{p}(1-\bar{p})}; \quad \mathrm{LCL_p} = \bar{p} - \mathrm{A}\sqrt{\bar{p}(1-\bar{p})}; \;\; \mathrm{CL_p} = \bar{p}$$

Problems for p-chart:

<u>EXAMPLE 1:</u>

Aim: The data given below are the number of defectives in 10 samples of 100 items each. Construct a p-chart and comment on the results.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	6	16	7	3	8	12	7	11	11	4

Find out the lower control limit and Upper control limit from data. Is the above data is within limits?

Procedure:

- Compute pi for each sample using the formula, $\mathbf{p_i} = \frac{\mathbf{d_i}}{\mathbf{n}}$.
  where n = number of observations.
- Calculate $\boldsymbol{\bar{p}}$ for the given data using the formula,

$$\bar{\mathrm{p}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{p_i}}{\mathrm{k}}, \bar{\mathrm{q}} = 1 - \bar{\mathrm{p}}$$

where k = number of defective.

- Calculate the control limits for p-chart using the formula,

$$\text{Upper control limit } \mathrm{UCL_p} = \overline{p} + 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}} \text{ and}$$

$$\text{Lower control limit } \mathrm{LCL_p} = \overline{p} + 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}}$$

$$\text{Center line } \mathrm{CL_p} = \overline{p}$$

- If LCL is negative, then consider it as zero.
- Plot the limits and Samples in the chart and interpret the result. Samples in the chart and interpret the result.

Calculation:

S.No.	d	p = d/100
1	6	0.06
2	16	0.16
3	7	0.07
4	3	0.03
5	8	0.08
6	12	0.12
7	7	0.07
8	11	0.11
9	11	0.11
10	4	0.04

$\sum p_i = 0.85$, k =10

$$\bar{p} = \sum_{i=1}^{n} \frac{p_i}{k} = \frac{0.85}{10} = 0.085$$

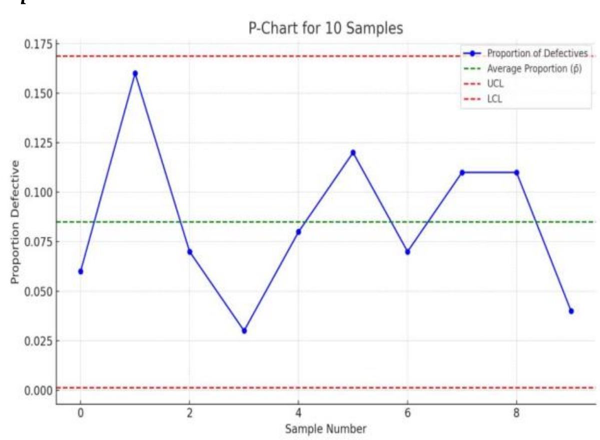
$$\bar{q} = 1 - \bar{p} = 1 - 0.085 = 0.915$$

p-chart:

Upper control limit $UCL_p = \overline{p} + 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}} = 0.085 + 3\sqrt{\frac{0.085(0.915)}{100}} = 0.169$

Lower control limit $LCL_p = \overline{p} - 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}} = 0.085 - 3\sqrt{\frac{0.085(0.915)}{100}} = 0.001$

Center line $CL_p = \overline{p} = 0.085$



Result:

As all specified pi values be inside the control limits, the process is in control.

(ii) Control chart for the number of defectives (np- Chart or d chart)

If instead of p, the sample proportion defective, we use d, the number of defectives in the sample, then the 3-$\boldsymbol{\sigma}$ control limits for d chart are given by

$$E(d) \pm 3S.E(d) = nP \pm 3\sqrt{nP(1-p)} = P \pm A\sqrt{PQ}$$

- **Case (I) Standards Specified.**
  If p' is the given or known value of P, then
  $$\mathrm{UCL_d} = \mathrm{np}' + 3\sqrt{nP'(1-P')};\quad \mathrm{LCL_d} = \mathrm{np}' - 3\sqrt{nP'(1-P')};\ \ \mathrm{CL_d} = \mathrm{np}'$$
- **Case (ii) standards not specified.**

  Using $\bar{p}$ as an estimate of P, we get
  $$\mathrm{UCL_d} = \mathrm{n}\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p}\,)};\quad \mathrm{LCL_d} = \mathrm{n}\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p}\,)};\ \ \mathrm{CL_d} = \mathrm{n}\bar{p}$$

Since p cannot be negative, if LCL as given by above formulae comes out to be negative, then it is taken to be zero.

(iii) Control chart for the number of defects per unit (c-chart)

The field of application of c chart is much more restricted as compared to $\bar{x}\ and\ R$ charts or *p* chart, An article which does not conform to one or more of the specifications, is termed as defective while any instance of article's lack of conformity to specifications is a defect. Thus, every defective contains one or more of the defects, e.g., a defective casting may further be examined for blow holes, cold shuts, rough surface, weak structure, etc.,

Unlike *d* or *np* chart which applies to the number of defectives in a sample, *c* chart applies to the number of defects per unit. Sample size for *c* chart may be a single unit like a radio, or group of units or it may be a unit of fixed time, length, area, etc.,

Application of c- chart

Although the applications of the c-chart are limited compared to the X, R, p and np-charts, yet a number of practical situations exist where we apply the c-chart. Some area where the c-chart is applied are listed below:

- C-chart. is the number of defects observed in a television, computer, laptop, mobile, etc.
- C-chart. is the number of defects observed in a woollen carpet, cloth, paper, etc.
- C-chart. is the number of air bubbles in a glass, bottle, paper weight, etc.
- C-chart. is the number of defects observed in aircraft engines.
- C-chart. is the number of breakdowns at weak spots in insulation in a given length of insulated wire.
- C-chart. is the number of imperfections observed in a bolt (large bundle) of cloth.

- C-chart. is the number of surface defects observed in a galvanised sheet, furniture, automobile, etc.
- The C-chart is also used in
  (a) Chemical laboratories,
  (b) Accident statistics both in highway accidents and industrial accidents.

Example 1:

A control chart is to be formed for a process in which laptops are produced. The inspection unit is one laptop and control chart for the number of defects is to be used. Preliminary data are recorded and 45 defects are found in 30 laptops. Obtain the control limits for the chart.

Solution:

Since we need to control the number of defects and the inspection unit is one laptop, we use the c-chart for number of defects. The average number of defects in the process is not given.

It is given that

The total number of defects in laptops = 45, and

The total number of laptops inspected (k) = 30.

$$\bar{C} = \frac{average\ number\ of\ defect}{number\ of\ items}$$

$$\bar{C} = \frac{1}{k}\sum_{i=1}^{k} c_i$$

$$\bar{C} = \frac{1}{30} \times 45 = 1.5$$

Calculate the centre line and control limits of the c-chart as follows:

$$\text{centre line CL} = \bar{C} = 1.5$$

$$\text{upper control limit UCL} = \bar{C} + 3\sqrt{\bar{C}} = 1.5 + 3\sqrt{1.5} = 5.175$$

$$\text{Lower control limit LCL} = \bar{C} - 3\sqrt{\bar{C}} = 1.5 - 3\sqrt{1.5} = -2.175 \sim 0$$

Control Charts For Number of Defects Per Unit (U-Chart)

- The procedure for drawing the u-chart for variable sample size is similar to the c-chart with constant size.
- The primary difference between the c-chart and u-chart is that instead of plotting the number of defects per sample, we plot the number of defects per item/unit and monitor them.
- Since the control limits are function of the sample size (n), these will vary with the sample size. So we should calculate the control limits separately for each sample.
- In such cases, the control limits are known as variable control limits.

Calculate the average sample size as follows:

$$\bar{n} = \frac{n_1 + n_2 + \cdots . + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i$$

Upper control limit $\mathrm{UCL_u} = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$

Lower control limit $\mathrm{LCL_u} = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$

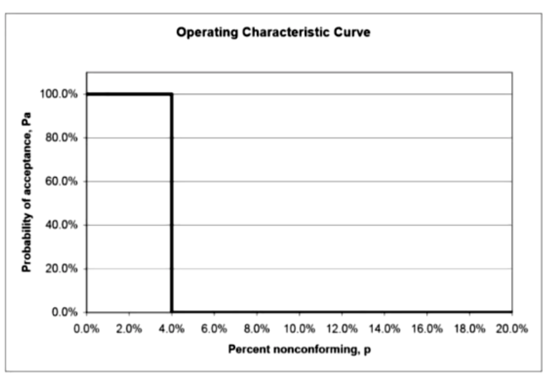
Center line $\mathrm{CL_u} = \bar{u}$

Operating Characteristic Curve

- Analysts create a graphic display of the performance of a sampling plan by plotting the probability of accepting the lot for a range of proportions of defective units.This graph, called an operating characteristic (OC) curve, describes how well a sampling plan discriminates between good and bad lots.
- Undoubtedly, every manager wants a plan that accepts lots with a quality level better than the AQL 100 percent of the time and accepts lots with a quality level worse than the AQL 0 percent of the time.
- However, such performance can be achieved only with 100 percent inspection. A typical OC curve for a single-sampling plan, plotted in red, shows the probability a of rejecting a good lot (producer's risk) and the probability of accepting a bad lot (consumer's risk).
- Consequently, managers are left with choosing a sample size n and an acceptance number to achieve the level of performance specified by the AQL, α, LTPD, and β.

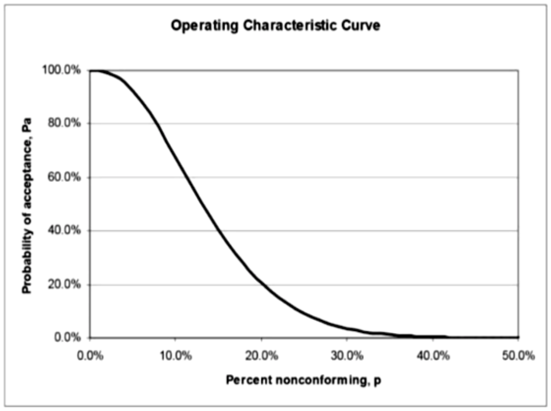
Ideal OC curve

- When percentage of non conforming items are below prescribed level probability of accepting is 100% and more than it makes probability of accepting 0%.
- Ideal ic curve can be obtained by 100% inspection
- Dividing line of probability of acceptance between 0 to 100% is AQL



Typical OC Curve:

- This is curve roughly S Shaped.
- Obtained by joining points between probability acceptance and percentage non conforming items.
- Obtained by performing sampling inspection.



Co-efficient of Variation (CV)

The co-efficient of variation (CV) is a statistical measure of the dispersion of data points in a data series around the mean. The co-efficient of variation represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different from one another.

Co-efficient of Variation (CV) Formula $\mathrm{c.v} = \frac{\sigma}{\mu}$

where: $\sigma = \text{standared deviation}$

$\mu = \text{mean}$

To calculate the CV for a sample, the formula is $c.v = \frac{s}{\bar{x}} \times 100$

where: s = sample and $\bar{x}$ = mean for the population

Advantages

- The co-efficient of variation can be useful when comparing data sets with different units or widely different means.
- That includes when the risk/reward ratio is used to select investments.
- For example, an investor who is risk-averse may want to consider assets with a historically low degree of volatility relative to the return, in relation to the overall market or its industry. Conversely, risk-seeking investors may look to invest in assets with a historically high degree of volatility.

Disadvantages

- When the mean value is close to zero, the CV becomes very sensitive to small changes in the mean.
- Using the example above, a notable flaw would be if the expected return in the denominator is negative or zero.
- In this case, the co-efficient of variation could be misleading.