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Unit-IV

Variables sampling plan

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Unit IV

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Variables sampling plan

This type of sampling plan is used when the quality of a product is measured on a continuous scale, such as weight or length.

For example, if a batch of products needs to have a specific weight range, variable sampling can be used to determine whether the batch meets this requirement or not.

Advantages and Disadvantages of Variables Sampling

Advantages	Disadvantages
The same OC curve can be obtained with a smaller sample size than would be required by an attributes sampling plan	Distribution of OC curve must be known Most standard plans assume distribution of quality characteristic is normal
Measurement data usually provide more information about manufacturing process than attributes data	A separate sampling plan must be employed for each quality characteristic that is being inspected.
When acceptable quality level as are very small, sample sizes required by attributes sampling plans are very large	Possible to reject a lot even though the actual sample inspected does not contain any defective items.

Single Sampling Plans for Variables

A brief description of single sampling plan for variables as described in Duncan (1986) when the observations measured are assumed to be normally distributed with known standard deviation σ is given below.

Let the quality characteristic X be measurable under continuous scale with a probability distribution N(μ,σ) and L be the specification limit so that an item is declared as defective if it fails to meet the specification namely, X<L or X>U where U be the upper specification limit.

The operating procedure of the single sampling plan for variables is defined as follows: Let $x_1, x_2, .. x_n$ be a random sample of n units drawn from a lot submitted for inspection. When σ is known, the lot is accepted if $\overline{x} + k\sigma \geq L$ otherwise the lot is rejected. As in the case of single sampling plan for attributes, the sampling plan for variables is designated by two parameters n (sample size) and k (acceptance criteria).

Chain Sampling

For situations in which testing is destructive or very expensive, sampling plans with small sample sizes are usually selected. These small sample size plans often have acceptance numbers of zero. Plans with zero acceptance numbers are often undesirable, however, in that Their OC curves are convex throughout.

This means that the probability of lot acceptance begins to drop very rapidly as the lot fraction defective becomes greater than zero. This is often unfair to the producer, and in situations where rectifying inspection is used, it can require the consumer to screen a large number of lots that are essentially of acceptable quality OC curves of sampling plans that have acceptance numbers of zero and acceptance numbers that are greater than zero.

Dodge (1955) suggested an alternate procedure, known as **chain sampling,** that might be a substitute for ordinary single-sampling plans with zero acceptance numbers in certain circumstances. Chain-sampling plans make use of the cumulative results of several preceding lots. The general procedure is as follows:

1. For each lot, select the sample of size *n* and observe the number of defectives.

2. If the sample has zero defectives, accept the lot; if the sample has two or more defectives, reject the lot; and if the sample has one defective, accept the lot provided there have been no defectives in the previous *i* lots.

Thus, for a chain-sampling plan given by $n = 5$, $i = 3$, a lot would be accepted if there were no defectives in the sample of five, or if there were one defective in the sample of five and no defectives had been observed in the samples from the previous three lots. This type of plan is known as a ChSP-1 plan.

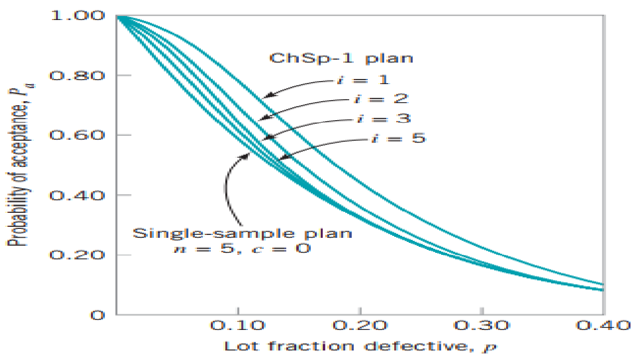
The effect of chain sampling is to alter the shape of the OC curve near the origin so that it has a more desirable shape. That is, it is more difficult to reject lots with very small fraction defectives with a ChSP-1 plan than it is with ordinary single sampling. Figure 16.6 shows OC curves for ChSP-1 plans with n = 5, c = 0, and i = 1, 2, 3, and 5. The curve for i = 1 is dotted, and it is not a preferred choice. In practice, values of *i* usually vary between three and five, since the OC curves of such plans approximate the single-sampling plan OC curve. The points on the OC curve of a ChSP-1 plan are given by the equation

$$P_a = P(0,n) + P(1,n)[P(0,n)]^i$$

Where $P(0, n)$ and $P(1, n)$ are the probabilities of obtaining 0 and 1 defectives, respectively, out of a random sample of size *n*. To illustrate the computations, consider the ChSP-1 plan with $n = 5$, $c = 0$, and $i = 3$. For $p = 0.10$, we have

$$P(0,n) = \frac{n!}{d!\,(n-d)!}p^d(1-p)^{n-d} = \frac{5!}{0!\,5!}(0.10)^0(0.90)^5 = 0.590$$

$$P(1,n) = \frac{n!}{d!\,(n-d)!}p^d(1-p)^{n-d} = \frac{5!}{1!\,(5-1)!}(0.10)^1(0.90)^4 = 0.328$$



And

$$P_a = P(0,n) + P(1,n)[P(0,n)]^i = 0.590 + (0.328)[0.590]^3 = 0.657$$

The proper use of chain sampling requires that the following conditions be met:

- **The** lot should be one of a series in a continuing stream of lots, from a process in which there is repetitive production under the same conditions, and in which the lots of products are offered for acceptance in substantially the order of production.
- Lots should usually be expected to be of essentially the same quality.
- The sampling agency should have no reason to believe that the current lot is of poorer quality than those immediately proceeding.
- There should be a good record of quality performance on the part of the supplier.
- The sampling agency must have confidence in the supplier, in that the supplier will not take advantage of its good record and occasionally send a bad lot when such a lot would have the best chance of acceptance.

Continuous Sampling

All the sampling plans discussed previously are lot-by-lot plans. With these plans, there is an explicit assumption that the product is formed into lots, and the purpose of the sampling plan is to sentence the individual lots. However, many manufacturing operations, particularly complex assembly processes, do not result in the natural formation of lots. For example, manufacturing of many electronics products, such as personal computers, is performed on a conveyor zed assembly line.

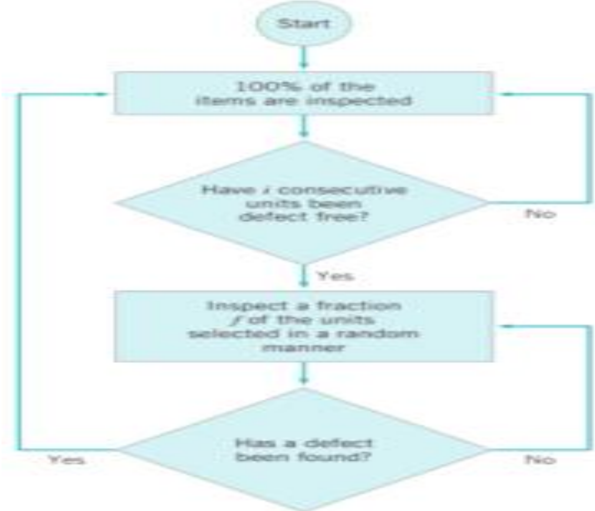
When production is continuous, two approaches may be used to form lots. The first procedure allows the accumulation of production at given points in the assembly process. This has the disadvantage of creating in-process inventory at various points, which requires additional space, may constitute a safety hazard, and is a generally inefficient approach to managing an assembly line.

The second procedure arbitrarily marks off a given segment of production as a "lot." The disadvantage of this approach is that if a lot is ultimately rejected and 100% inspection of the lot is subsequently required, it may be necessary to recall products from manufacturing operations that are further downstream. This may require disassembly or at least partial destruction of semi finished items. For these reasons, special sampling plans for continuous production have been developed.

Continuous-sampling plans consist of alternating sequences of sampling inspection and screening (100% inspection). The plans usually begin with 100% inspection, and when a stated number of units is found to be free of defects (the number of units i is usually called the **clearance number**), sampling inspection is instituted. Sampling inspection continues until a specified number of defective units is found, at which time 100% inspection is resumed. Continuous-sampling plans are rectifying inspection plans, in that the quality of the product is improved by the partial screening.

CSP-1

Continuous-sampling plans were first proposed by Harold F. Dodge (1943). Dodge's initial plan is called CSP-1. At the start of the plan, all units are inspected 100%. As soon as the clearance number has been reached—that is, as soon as *i* consecutive units of product are found to be free of defects—100% inspection is discontinued, and only a fraction (*f*) of the units are inspected. These sample units are selected one at a time at random from the flow of production. If a sample unit is found to be defective, 100% inspection is resumed. All defective units found are either reworked or replaced with good ones. The procedure for CSP-1 is shown in



A CSP-1 plan has an overall AOQL. The value of the AOQL depends on the values of the clearance number *i* and the sampling fraction *f*. The same AOQL can be obtained by different combinations of *i* and *f*. presents various values of *i* and *f* for CSP-1 that will lead to a stipulated AOQL. Note in the table that an AOQL of 0.79% could be obtained using a sampling plan with *i* = 59 and or with *i* = 113 and the choice of *i* and *f* is usually based on practical considerations in the manufacturing process.

For example, *i* and *f* may be influenced by the workload of the inspectors and operators in the system. It is a fairly common practice to use quality-assurance inspectors to do the sampling inspection, and place the burden of 100% inspection on manufacturing. As a general rule, however, it is not a good idea to choose values of *f* smaller than 1/200 because the protection against bad quality in a continuous runs of production then becomes very poor.

The average number of units inspected in a 100% screening sequence following the occurrence of a defect is equal to

$$u = \frac{1 - q^i}{pq^i}$$

TABLE 16.3
Values of *i* for CSP-1 Plans

	AOQL (%)															
f	0.018	0.033	0.046	0.074	0.113	0.143	0.198	0.33	0.53	0.79	1.22	1.90	2.90	4.94	7.12	11.46
1/2	1,540	840	600	375	245	194	140	84	53	36	23	15	10	6	5	3
1/3	2,550	1,390	1,000	620	405	321	232	140	87	59	38	25	16	10	7	5
1/4	3,340	1,820	1,310	810	530	420	303	182	113	76	49	32	21	13	9	6
1/5	3,960	2,160	1,550	965	630	498	360	217	135	91	58	38	25	15	11	7
1/7	4,950	2,700	1,940	1,205	790	623	450	270	168	113	73	47	31	18	13	8
1/10	6,050	3,300	2,370	1,470	965	762	550	335	207	138	89	57	38	22	16	10
1/15	7,390	4,030	2,890	1,800	1,180	930	672	410	255	170	108	70	46	27	19	12
1/25	9,110	4,970	3,570	2,215	1,450	1,147	828	500	315	210	134	86	57	33	23	14
1/50	11,730	6,400	4,590	2,855	1,870	1,477	1,067	640	400	270	175	110	72	42	29	18
1/100	14,320	7,810	5,600	3,485	2,305	1,820	1,302	790	500	330	215	135	89	52	36	22
1/200	17,420	9,500	6,810	4,235	2,760	2,178	1,583	950	590	400	255	165	106	62	43	26

Where q = 1 - p, and p is the fraction defective produced when the process is operating in control. The average number of units passed under the sampling inspection procedure before a defective unit is found is

$$v = \frac{1}{fp}$$

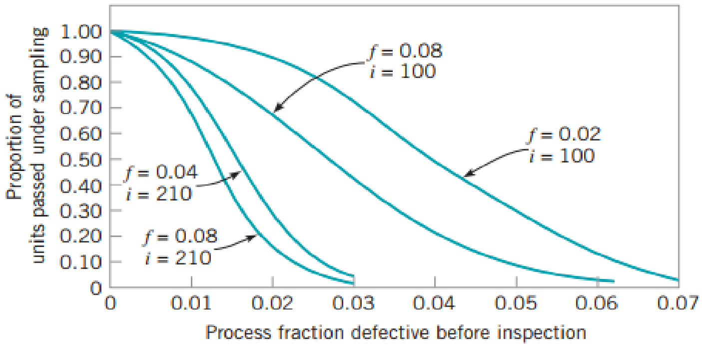
The average fraction of total manufactured units inspected in the long run is

$$AFI = \frac{u + fv}{u + v}$$

The average fraction of manufactured units passed under the sampling procedure is

$$P_u = \frac{v}{u + v}$$

When *Pa* is plotted as a function of *p*, we obtain an operating-characteristic curve for a continuous-sampling plan. Note that whereas an OC curve for a lot-by-lot acceptance-sampling plan gives the percentage of lots that would be passed under sampling inspection; the OC curve for a continuous-sampling plan gives the percentage of units passed under sampling inspection. Graphs of operating-characteristic curves for several values of *f* and *i* for CSP-1 plans are shown in Fig. 16.8. Note that for moderate-to-small values of *f*, *i* have much more effect on the shape of the curve than does *f*



Assumptions

1. All defectives found during inspection are rectified or replaced by good items.
2. Inspection is perfect, i.e. mistakes in identifying defectives are never made.
3. Theoretical calculations are made on the assumption that the process is producing defectives with probability p, and that the probability that any item is defective is independent of the quality of other items.

Modifications to CSP-1

Over the years various modifications have been suggested to CSP-1. Dodge and Torrey (1951) suggested the following two plans:

CSP-2

CSP-2 Proceed as in CSP-l except that, once partial inspection is instituted, 100% inspection is only introduced when two defectives occur spaced less than k items a part. This plan is less likely to revert to 100% inspection because of isolated defectives than is the CSP-1, and the number of abrupt changes of inspection level will also be reduced. However, there is a higher risk of accepting short runs of poor quality, and so CSP-3 is suggested.

CSP-3

CSP-3 Proceed as in CSP-2 except that when a defective is found, the next four items are inspected. The theory of these two plans follows a similar pattern to the theory for CSP-1,

$$(i+1)p_1 - 1 = (n-1)(1-p_1)^{i+1}$$

Although in each case it is more complicated. Another line of development attempts to devise plans which guarantee an AOQL without assuming statistical control of the process. The starting-point of these investigations is a. paper by Lieberman (1953), who examined the AOQL of the CSP-1 without the assumption of control. It is not difficult to see that this is attained by a process which produces good items throughout periods of 100% inspection, and defectives throughout periods of partial inspection. Periods of 100% inspection are therefore exactly i items long, and the average number of items produced between the start of such periods is (n + i).

One defective item will be inspected, and consequently replaced by a good item. The average fraction defective remaining after inspection is therefore (n- 1)/(n + i), which can be considerably greater than (5.2). For a formal proof of this formula, see Derman et al. (1959). When interpreting this result, however, it is important to take note of the pathological nature of the production process model which produces it. Derman et al. (1959) present two variants of CSP-1 which have improved properties when control is not assumed.

Wald–Wolfowitz Runs test

- The runs test is a shortened version of the full name: the Wald–Wolfowitz runs test, so named after mathematicians Abraham Wald and Jacob Wolfowitz.
- A runs test is a statistical procedure that examines whether a sequence of data is occurring randomly from a specific distribution. That is run test is used for examining whether or not a set of observations constitutes a random sample from an infinite population. Test for randomness of major importance because the assumption of randomness underlies statistical inference.
- The run test for randomness is carried out in a random model in which the observations vary around a constant mean. The observation in the random model in which the run test is carried out has a constant variance, and the observations are also probabilistically independent.

Understanding a Run

- A run is defined as a series of increasing values or a series of decreasing values. The number of increasing, or decreasing, values is the length of the run. In short a run and the length of the run can be defined as
- Run – sequence of similar events, items, or symbols that is followed by an event, item, or symbol that is mutually exclusive from the first event, item, or symbol
- Length – number of events, items, or symbols in a run
- For example, a series of 20 coin tosses might produce the following sequence of heads (H) and tails (T).
- H H T T H T H H H H T H H T T T T T H H
- The number of runs for this series is nine. There are 11 heads and 9 tails in the sequence.

One Sample Wald–Wolfowitz Runs test

- In this case the Null hypothesis and Alternative hypothesis: are

- H_0 : Sample value come from a random Sequence (or the given Sequence is random)
- H_1: Sample value come from a non-random Sequence (or the given Sequence is non-random)
- Test Statistics is the number of runs r
- For finding the number of runs, the observations are listed in their order of occurrence. Each observation is denoted by '+' signs if it is more than the previous observation by a '-' sign if it is less than the previous observation. If the observation is same as previous observation put '0'. The total number of runs up (+) and down (-) is counted.
- Criteria value for test is obtained from the table for wald- wolfowitz runs test one sample for a given value of n and desired level of significanceα. Let this value be r_{cl}(lower value) and r_{cu}(upper value).
- Decision criteria is
  Accept H_0 if $r_{cl} \leq$ r $\leq r_{cu.}$ otherwise rejectH_0.
- For large Sample: When sample size is greater than 25, the critical values r_{cl} and $r_{cu}$is obtained using normal approximation as $r_{cl} = \mu - 1.96\sigma$ and $r_{cu} = \mu + 1.96\sigma$ at 5% level of significant. Where

$$\mu = \frac{2n-1}{3} \; and \; \sigma = \sqrt{\frac{16n-29}{90}}.$$

Two samples Wald–Wolfowitz Runs test

- Wald-Wolfowitz Two-sample Run test is used to examine whether two random samples came from population having same distribution. This test can detect differences in average or spread or any other important aspect between the two populations.
- Hypothesis
- H_0 : Sample value came from the population having same distribution
- H_1 : Sample value came from the population having different distribution
- Test Statistics is the number of runs r.
- Let r denote the number of runs. To obtain r, list the $n_1 + n_2$ observations from two samples in ascending order of magnitude. Denote observations from one sample as x's and other by y's. Count the number of runs. In case x and y observation are same value, place the observation x(y) first if run of x(y) observation is counting.
- Critical value for test is obtained from the table for Wald–Wolfowitz runs test two samples for a given value of n_1 and n_2. desired level of significance α. Let this value be a r_c note that this one sided hypothesis.
- Decision criteria is
  Accept H_0 if $r > r_c$. or reject$H_0 if \; r < r_{c.}$
- For large Sample: When Sample size is greater than 25, the critical values r_c is obtained using normal approximation as $r_c = \mu - 1.96\sigma$ at 5% level of significant. Where

$$\mu = 1 + \frac{2n_1 n_2}{n_1 + n_2} \; and$$

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

Problem:

There are two IVM (Innoson Vehicle Manufacturers) buses, one with 48 passengers, and another with 38 passengers. Let X and Y denote the number of miles travelled per day for the 48-passenger and 38-passenger buses respectively. Innoson would like to test the equality of the two distributions.

That is, if:

$$H_0: F(z) = G(z)$$

The company observed the following data on a random sample of $n_1 = 10$ buses carrying 48 passengers and $n_2 = 11$ buses carying 38 passengers.

X: 104 253 300 308 315 323 331 396 414 452

Y: 184 196 197 248 260 279 355 386 393 432 450

Solution

Step 1: State the null and the alternate hypothesis and rejection criteria

$$H_0: F(z) = G(z)$$
$$H_1: F(z) = G(z)$$

Rejection criteria: Reject the null hypothesis if

$$Z \leq -z_\alpha$$

Step 2: Merge the two lists and sort in ascending order

104 184 196 197 248 253 260 279 300 308 315 331 355 386 393 394 414 432 450 452

Step 3: Count the number of runs: R, n_1 and n_2

Number of runs R = 9

$n_1 = 10$

$n_2 = 11$

Step 4: Calculate the mean

We calculate the mean using the formula and we have the results below

$$\mu = 1 + \frac{2n_1n_2}{n_1+n_2}$$

$$= 1 + \frac{2(10)(11)}{10+11}$$

$$= 1 + \frac{220}{21}$$

$$\mu = 11.476$$

Step 5: Calculate the variance

We calculation the variance using the calculation steps below

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$\sigma = \sqrt{\frac{2(10)(11)(2 \times 10 \times 11 - 10 - 11)}{(10+11)^2(10+11-1)}}$$

$$= \sqrt{\frac{220(220-21)}{(21)^2(20)}}$$

$$= \sqrt{4.9637}$$

$$\sigma = 2.2279$$

Step 6: Calculate Z

We calculate the value of Z following the formula below:

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{9.5 - 11.476}{\sqrt{4.9367}} = -0.89$$

Step 7: Draw your conclusion

We fail to reject the null hypothesis at the 0.05 level because the P value is greater than 0.05. This means that there is not sufficient evidence at 0.05 level to conclude that the two distribution functions are not equal.

Skip-Lot Sampling Plans

This section describes the development and evaluation of a system of lot-by-lot inspection plans in which a provision is made for inspecting only some fraction of the submitted lots. These plans are known as **skip-lot sampling plans.** Generally speaking, skip-lot sampling plans should be used only when the quality of the submitted product is good as demonstrated by the supplier’s quality history.

Dodge (1956) initially presented skip-lot sampling plans as an extension of CSP-type continuous-sampling plans. In effect, a skip-lot sampling plan is the application of continuous sampling to lots rather than to individual units of production on an assembly line. The version of skip-lot sampling initially proposed by Dodge required a single determination or analysis to ascertain the lot’s acceptability or unacceptability. These plans are called SkSP-1. Skip-lot sampling plans designated SkSP-2 follow the next logical step; that is, each lot to be sentenced is sampled according to a particular attribute lot inspection plan. Perry (1973) gives a good discussion of these plans.

A skip-lot sampling plan of type SkSP-2 uses a specified lot inspection plan called the “reference-sampling plan,” together with the following rules:

- Begin with normal inspection, using the reference plan. At this stage of operation, every lot is inspected.
- When *i* consecutive lots are accepted on normal inspection, switch to skipping inspection. In skipping inspection, a fraction *f* of the lots is inspected.
- When a lot is rejected on skipping inspection, return to normal inspection the parameters *f* and *i* are the parameters of the skip-lot sampling plan SkSP-2. In general,

the clearance number *i* is a positive integer, and the sampling fraction *f* lies in the interval $0 < f < 1$. When the sampling fraction $f = 1$, the skip-lot sampling plan reduces to the original reference-sampling plan. Let *P* denote the probability of acceptance of a lot from the reference-sampling plan. Then, *Pa* (*f*, *i*) is the probability of acceptance for the skip-lot sampling plan SkSP-2,

Where

$$P_a(f,i) = \frac{fP + (1-f)P^i}{f + (1+f)P^i}$$

It can be shown that for $f_2 < f_1$, a given value of the clearance number *i*, and a specified reference-sampling plan,

$$P_a(f_1, i) \le P_a(f_2, i)$$

Furthermore, for integer clearance numbers i<j, a fixed value of f, and a given reference sampling plan.

$$P_a(f, j) \le P_a(f, j)$$

These properties of a skip-lot sampling plan are shown in Figs. 16.9 and 16.10 for the reference-sampling plan $n = 20$, $c = 1$. The OC curve of the reference-sampling plan is also shown on these graphs.

A very important property of a skip-lot sampling plan is the average amount of inspection required. In general, skip-lot sampling plans are used where it is necessary to reduce the average amount of inspection required. The average sample number of a skip-lot sampling plan is

ASN (SkSP)=ASN(R)F

Where F is the average fraction of submitted lots that are sampled and ASN(R) is the average Sample number of the reference sampling plan. It can be shown that

$$F = \frac{f}{(1-f)P^i + f}$$

Thus, since $0 < F < 1$, it follows that

ASN (SkSP) < ASN(R)

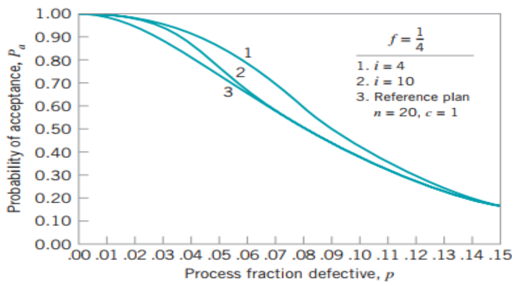


FIGURE 16.9 OC curves for SkSP-2 skip-lot plans: single-sampling reference plan, same *f*, different *i*. (From R. L. Perry, "Skip-Lot Sampling Plans," *Journal of Quality Technology*, Vol. 5, 1973, with permission of the American Society for Quality Control.)

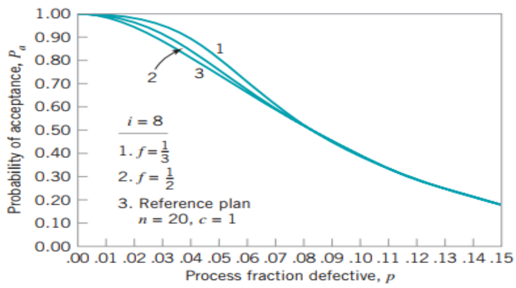


FIGURE 16.10 OC curves for SkSP-2 skip-lot plans: single-sampling reference plan, same *i*, different *f*. (From R. L. Perry, "Skip-Lot Sampling Plans," *Journal of Quality Technology*, Vol. 5, 1973, with permission of the American Society for Quality Control.)

Therefore, skip-lot sampling yields a reduction in the **average sample number (ASN).** For situations in which the quality of incoming lots is very high, this reduction in inspection effort can be significant. To illustrate the average sample number behaviour of a skip-lot sampling plan, consider a reference-sampling plan of $n = 20$ and $c = 1$. Since the average sample number for a single sampling plan is ASN = *n*, we have

ASN (SkSP)=n(F)

presents the ASN curve for the reference-sampling plan *n* = 20, *c* = 1 and the following skip-lot sampling plans:

- $f = \frac{1}{3}, i = 4$
- $f = \frac{1}{3}, i = 14$
- $f = \frac{2}{3}, i = 4$
- $f = \frac{2}{3}, i = 14$

For small values of incoming lot fraction defective, the reductions in average sample number are very substantial for the skip-lot sampling plans evaluated. If the incoming lot quality is very good, consistently close to zero fraction nonconforming, say, then a small value of *f*, perhaps or, could be used. If incoming quality is slightly worse, then an appropriate value of *f* might be Skip-lot sampling plans are an effective acceptance-sampling procedure and may be useful as a system of reduced inspection.

Their effectiveness is partially good when the quality of submitted lots is very good. However, one should be careful to use skip-lot sampling plans only for situations in which there is a sufficient history of supplier quality to ensure that the quality of submitted lots is very good. Furthermore, if the supplier's process is highly erratic and there is a great deal of variability from lot to lot, skip-lot sampling plans are inappropriate. They seem to work best when the supplier's processes are in a state of statistical control and when the process capability is adequate to ensure virtually defect-free production.

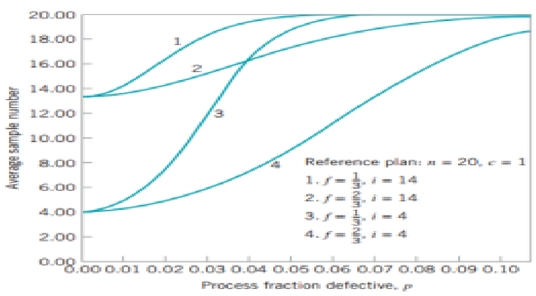


FIGURE 16.11 Average sample number (ASN) curves for SkSP 2 skip-lot plans with single-sampling reference plan. (From R. L. Perry, "Skip-Lot Sampling Plans," *Journal of Quality Technology*, Vol. 5, 1973, with permission of the American Society for Quality Control.)

Six Sigma

Six Sigma is a set of methodologies and tools used to improve business processes by reducing defects and errors, minimizing variation, and increasing quality and efficiency. The goal of Six Sigma is to achieve a level of quality that is nearly perfect, with only 3.4 defects per million opportunities. This is achieved by using a structured approach called DMAIC (Define, Measure, Analyze, Improve, and Control) to identify and eliminate causes of variation and improve processes.

Six sigma is a disciplined and data-driven approach widely used in project management to achieve process improvement and minimize defects. It provides a systematic framework to identify and eliminate variations that can impact project performance.

The etymology is based on the Greek symbol "sigma" or "σ," a statistical term for measuring process deviation from the process mean or target. "Six Sigma" comes from the bell curve used in statistics, where one Sigma symbolizes a single standard deviation from the mean. If the process has six Sigma's, three above and three below the mean, the defect rate is classified as "extremely low."

The graph of the normal distribution below underscores the statistical assumptions of the Six Sigma model. The higher the standard deviation, the higher is the spread of values encountered. So, processes, where the mean is minimum 6σ away from the closest specification limit, are aimed at Six Sigma.

This success led to many organizations adopting the approach. Since its origins, there have been three generations of six-sigma implementations.

Generation I

Six-sigma focused on defect elimination and basic variability reduction. Motorola is often held up as an exemplar of Generation I six-sigma.

Generation II

Six-sigma, the emphasis on variability and defect reduction remained, but now there was a strong effort to tie these efforts to projects and activities that improved business performance through cost reduction. General Electric is often cited as the leader of the Generation II phase of six-sigma

Generation III

Six-sigma has the additional focus of creating value throughout the organization and for its stakeholders (owners, employees, customers, suppliers, and society at large). Creating value can take many forms: increasing stock prices and dividends, job retention or expansion, expanding markets for company products/services, developing new products/services that reach new and broader markets, and increasing the levels of customer satisfaction throughout the range of products and services offered.

These are strong endorsements of six-sigma from two highly recognized business leaders that lead two very different types of organizations, manufacturing and financial services. Caterpillar and Bank of America are good examples of Generation III six-sigma companies, because their implementations are focused on value creation for all stakeholders in the broad sense.

Note Lewis's emphasis on reducing cycle times and reducing processing errors (items that will greatly improve customer satisfaction), and Owens's remarks on extending six-sigma to suppliers and dealers—the entire supply chain. Six-sigma has spread well beyond its manufacturing origins into areas including health care, many types of service business, and government/public service (the U.S. Navy has a strong and very successful six-sigma program).

The reason for the success of six-sigma in organizations outside the traditional manufacturing sphere is that variability is everywhere, and where there is variability, there is an opportunity to improve business results. Some examples of situations where a six-sigma program can be applied to reduce variability, eliminate defects, and improve business performance include:

- Meeting delivery schedule and delivery accuracy targets
- Eliminating rework in preparing budgets and other financial documents
- Proportion of repeat visitors to an e-commerce Website, or proportion of visitors that make a purchase
- Minimizing cycle time or reducing customer waiting time in any service system
- Reducing average and variability in days outstanding of accounts receivable
- Optimizing payment of outstanding accounts
- Minimizing stock-out or lost sales in supply chain management
- Minimizing costs of public accountants, legal services, and other consultants
- Inventory management (both finished goods and work-in-process)
- Improving forecasting accuracy and timing
- Improving audit processes
- Closing financial books, improving accuracy of journal entry and posting (a 3 to 4% error rate is fairly typical)
- Reducing variability in cash flow
- Improving payroll accuracy
- Improving purchase order accuracy and reducing rework of purchase orders

Lean of Six Sigma

Lean Six Sigma is a methodology that combines two powerful process improvement techniques: Lean and Six Sigma. Lean focuses on minimizing waste and maximizing efficiency by identifying and eliminating non-value-adding activities. This involves streamlining processes, reducing defects, improving quality, and optimizing resources to deliver more value with less effort.

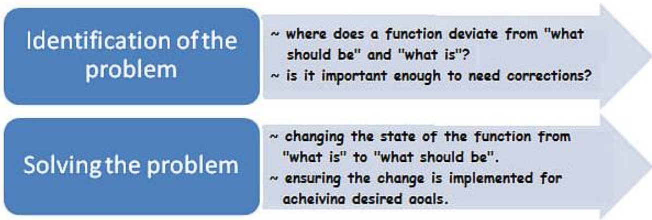
On the other hand, Six Sigma is a statistical approach to process improvement that aims to reduce variation and defects by using data-driven decision making. It involves defining, measuring, analyzing, improving, and controlling processes to achieve consistent and predictable results.

By combining the strengths of these two methodologies, Lean Six Sigma provides a comprehensive approach to process improvement that can be applied to any industry or sector. It is widely used in manufacturing, healthcare, finance, and service industries to improve efficiency, reduce costs, and enhance customer satisfaction.

Principles of Six Sigma

The concept of Six Sigma has a simple goal – delivering near-perfect goods and services for business transformation for optimal customer satisfaction (CX).

Goals are achieved through a two-pronged approach:



Six Sigma has its foundations in five key principles:

1. Focus on the Customer

   This is based on the popular belief that the "customer is the king." The primary goal is to bring maximum benefit to the customer. For this, a business needs to understand its customers, their needs, and what drives sales or loyalty. This requires establishing the standard of quality as defined by what the customer or market demands.
2. Measure the Value Stream and Find Your Problem

   Map the steps in a given process to determine areas of waste. Gather data to discover the specific problem area that is to be addressed or transformed. Have clearly defined goals for data collection, including defining the data to be collected, the reason for the data gathering, insights expected, ensuring the accuracy of measurements, and establishing a standardized data collection system. Ascertain if the data is helping to achieve the goals, whether or not the data needs to be refined, or additional information collected. Identify the problem. Ask questions and find the root cause.
3. Get Rid of the Junk

   Once the problem is identified, make changes to the process to eliminate variation, thus removing defects. Remove the activities in the process that do not add to the customer value. If the value stream doesn't reveal where the problem lies, tools are used to help discover the outliers and problem areas. Streamline functions to achieve quality control and efficiency. In the end, by taking out the above-mentioned junk, bottlenecks in the process are removed.
4. Keep the Ball Rolling

   Involve all stakeholders. Adopt a structured process where your team contributes and collaborates their varied expertise for problem-solving. Six Sigma processes can have a great impact on an organization, so the team has to be proficient in the principles and methodologies used. Hence, specialized training and knowledge are required to reduce the risk of project or re-design failures and ensure that the process performs optimally.
5. Ensure a Flexible and Responsive Ecosystem

   The essence of Six Sigma is business transformation and change. When a faulty or inefficient process is removed, it calls for a change in the work practice and employee approach. A robust culture of flexibility and responsiveness to changes in procedures can ensure streamlined project implementation. The people and departments involved should be able to adapt to change with ease, so to facilitate this, processes should be designed for quick and seamless adoption. Ultimately, the company that has an eye fixed on the data examines the bottom line periodically and adjusts its processes where necessary, can gain a competitive edge.

Methodology

The two main Six Sigma methodologies are DMAIC and DMADV. Each has its own set of recommended procedures to be implemented for business transformation.

DMAIC is a data-driven method used to improve existing products or services for better customer satisfaction. It is the acronym for the five phases: D – Define, M – Measure, A – Analyse, I – Improve, C – Control. DMAIC is applied in the manufacturing of a product or delivery of a service.

DMADV is a part of the Design for Six Sigma (DFSS) process used to design or re-design different processes of product manufacturing or service delivery. The five phases of DMADV are: D – Define, M – Measure, A – Analyse, D – Design, V – Validate. DMADV is employed when existing processes do not meet customer conditions, even after optimization, or when it is required to develop new methods. It is executed by Six Sigma Green Belts and Six Sigma Black Belts and under the supervision of Six Sigma Master Black Belts. We'll get to the belts later.

The two methodologies are used in different business settings, and professionals seeking to master these methods and application scenarios would do well to take an online certificate program taught by industry experts. The Six Sigma Process of the DMAIC method has five phases: Each of the above phases of business transformation has several steps:

DEFINE

The Six Sigma process begins with a customer-centric approach.

Step 1: The business problem is defined from the customer perspective.

Step 2: Goals are set. What do you want to achieve? What are the resources you will use to achieve the goals?

Step 3: Map the process. Verify with the stakeholders that you are on the right track.

MEASURE

The second phase is focused on the metrics of the project and the tools used in the measurement. How can you improve? How can you quantify this?

Step 1: Measure your problem in numbers or with supporting data.

Step 2: Define performance yardstick. Fix the limits for "Y."

Step 3: Evaluate the measurement system to be used. Can it help you achieve your outcome?

ANALYZE

The third phase analyzes the process to discover the influencing variables.

Step 1: Determine if your process is efficient and effective. Does the process help achieve what you need?

Step 2: Quantify your goals in numbers. For instance, reduce defective goods by 20%.

Step 3: Identify variations using historical data.

IMPROVE

This process investigates how the changes in "X" impact "Y." This phase is where you identify how you can improve the process implementation.

Step 1: Identify possible reasons. Test to identify which of the "X" variables identified in Process III influence "Y".

Step 2: Discover relationships between the variables.

Step 3: Establish process tolerance, defined as the precise values that certain variables can have, and still fall within acceptable boundaries, for instance, the quality of any given product. Which boundaries need X to hold Y within specifications? What operating conditions can impact the outcome? Process tolerances can be achieved by using tools like robust optimization and validation set.

CONTROL

In this final phase, you determine that the performance objective identified in the previous phase is well implemented and that the designed improvements are sustainable.

Step 1: Validate the measurement system to be used.

Step 2: Establish process capability. Is the goal being met? For instance, will the goal of reducing defective goods by 20 percent be achieved?

Step 3: Once the previous step is satisfied, implement the process

Comparison of DMAIC and DMADV

DMAIC	DMADV
DEFINE Define the project goals and customer deliverables	DEFINE Define the project goals and customer deliverables.
MEASURE Measure the process to determine the current performance	MEASURE Measure and determine the customer needs and specifications
ANALYSE Analyse and determine the root cause of defect	ANALYSE Analyse the product or process options to meet customer needs.
IMPROVE Improve the process by the eliminating defects.	DESIGN Design (detailed) the product or process to meet customer needs.
CONTROL Control future process performance	VERIFY Verify the design performance and ability to meet customer needs.

Uses of DMAIC

DMAIC methodology should be used when a product or process is in existence at your company but it is not meeting customer specifications of is not performing adequately.

Uses of DMADV

DMADV methodology should be used when

- A product or process is not in existence at your company and one needs to be developed.
- The product or process exists and has been optimized (using either DMAIC or not) and still does not meet the level of customer specification or six sigma level.

Types of data and graphs used in Six sigma

Six Sigma uses different types of data and graphs. Let us discuss about this in detail in this section.

Mainly, data are of two types

- Continuous data

  Continuous data is a type of data which can be measured it is value on a continuum. For example, inches, centimetres, pounds, kilogram, etc, value can be in decimal or fraction form also.

- Discrete data

  Discrete data is a type of data which can be counted. Values like integers, whole number. They are also called categorical data.

  Data types impact the type of graph that can be used. For continuous data graphs can be used to variations.

  Continuous data and graphs and charts include;

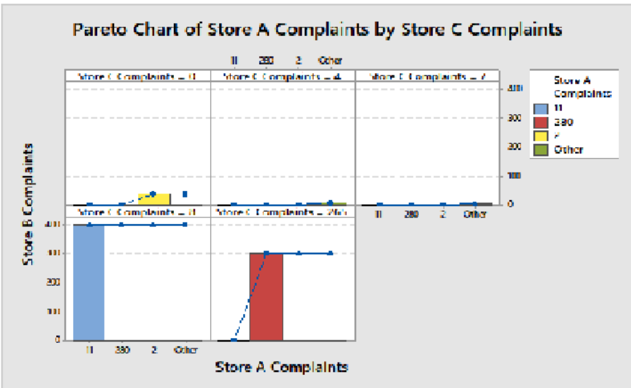
  - ✓ Histogram
  - ✓ Dot plots
  - ✓ Box plots

  Discrete or categorical data graphs and chart include;

  - ✓ Bar chart
  - ✓ Pareto charts

  Graphs showing trends over time are;

  - ✓ Line graph
  - ✓ Time series charts
  - ✓ Control charts



Six Sigma Techniques

The Six Sigma methodology also uses a mix of statistical and data analysis tools such as process mapping and design and proven qualitative and quantitative techniques, to achieve the desired outcome.

A. Brainstorming
B. Root Cause Analysis/The 5 Whys
C. Voice of the Customer
D. The 5S System
E. Kaizen (Continuous Improvement)

F. Benchmarking
G. Poka-yoke (Mistake Proofing)
H. Value Stream Mapping

A. Brainstorming

Brainstorming is the key process of any problem-solving method and is often utilized in the "improve" phase of the DMAIC methodology. It is a necessary process before anyone starts using any tools. Brainstorming involves bouncing ideas and generating creative ways to approach a problem through intensive freewheeling group discussions. A facilitator, who is typically the lead Black Belt or Green Belt, moderates the open session among a group of participants.

B. Root Cause Analysis/ The 5 Whys

This technique helps to get to the root cause of the problems under consideration and is used in the "analyze" phase of the DMAIC cycle.

In the 5 Whys technique, the question "why" is asked, again and again, finally leading up to the core issue. Although "five" is a rule of thumb, the actual number of questions can be greater or fewer, whatever it takes to gain clarity.

C. Voice of the Customer

This is the process used to capture the "voice of the customer" or customer feedback by either internal or external means. The technique is aimed at giving the customer the best products and services. It captures the changing needs of the customer through direct and indirect methods. The voice of the customer technique is used in the "define' phase of the DMAIC method, usually to further define the problem to be addressed.

D. The 5S System

This technique has its roots in the Japanese principle of workplace energies. The 5S System is aimed at removing waste and eliminating bottlenecks from inefficient tools, equipment, or resources in the workplace. The five steps used are Seiri (Sort), Seiton (Set In Order), Seiso (Shine), Seiketsu (Standardize), and Shitsuke (Sustain).

E. Kaizen (Continuous Improvement)

The Kaizen technique is a powerful strategy that powers a continuous engine for business improvement. It is the practice continuously monitoring, identifying, and executing improvements. This is a particularly useful practice for the manufacturing sector. Collective and ongoing improvements ensure a reduction in waste, as well as immediate change whenever the smallest inefficiency is observed.

F. Benchmarking

Benchmarking is the technique that employs a set standard of measurement. It involves making comparisons with other businesses to gain an independent appraisal of the given situation. Benchmarking may involve comparing important processes or departments within a business (internal benchmarking), comparing similar work areas or functions with industry leaders (functional benchmarking), or comparing similar products and services with that of competitors (competitive benchmarking).

G. Poka-yoke (Mistake Proofing)

This technique's name comes from the Japanese phrase meaning "to avoid errors," and entails preventing the chance of mistakes from occurring. In the poka-yoke technique, employees spot and remove inefficiencies and human errors during the manufacturing process.

H. Value Stream Mapping

The value stream mapping technique charts the current flow of materials and information to design a future project. The objective is to remove waste and inefficiencies in the value stream and create leaner operations. It identifies seven different types of waste and three types of waste removal operations.

The Six Sigma Tools

1. Cause and Effect Analysis
2. Flow Chart
3. Pareto Chart
4. Histogram
5. Check Sheet
6. Scatter Plot
7. Control Chart.

Six Sigma Levels

The Six Sigma training levels conform to specified training requirements, education criteria, job standards, and eligibility.

White Belt

This is the simplest stage, where:

- Any newcomer can join.
- People work with teams on problem-solving projects.
- The participant is required to understand the basic Six Sigma concepts.

Yellow Belt

Here, the participant:

- Takes part as a project team member.
- Reviews process improvements.
- Gains understanding of the various methodologies, and DMAIC.

Green level

This level of expertise requires the following criteria:

- Minimum of three years of full-time employment.
- Understand the tools and methodologies used for problem-solving.
- Hands-on experience on projects involving some level of business transformation.
- Guidance for Black Belt projects in data collection and analysis.
- Lead Green Belt projects or teams.

Black Level

This level includes the following:

- Minimum of three years of full-time employment
- Work experience in a core knowledge area
- Proof of completion of a minimum of two Six Sigma projects
- Demonstration of expertise at applying multivariate metrics to diverse business change settings
- Leading diverse teams in problem-solving projects.
- Training and coaching project teams.

Master Black Belt
To reach this level, a candidate must:

- Be in possession of a Black Belt certification
- Have a minimum of five years of full-time employment, or Proof of completion of a minimum of 10 Six Sigma projects
- A proven work portfolio, with individual specific requirements, as given here, for instance.
- Have coached and trained Green Belts and Black Belts.
- Develop key metrics and strategies.
- Have worked as an organization's Six Sigma technologist and internal business transformation advisor.

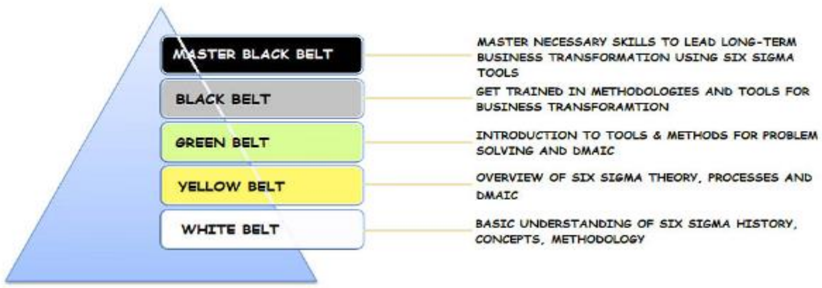


Fig: The five-tiered levels of Six Sigma Certification

Advantages of the six sigma process

There are many advantages to using the Six Sigma process. By using this method, leaders introduce a system of continuous process refinements, which small and big businesses can use with multiple applications.

For instance, an HR department may determine its losing millions of dollars because of high employee turnover. By implementing Six Sigma, the department could identify what’s causing the problem. They may identify low compensation and poor employee selection.

To improve the hiring process, they introduce more training for employees and a better compensation plan. Through Six Sigma, the HR department can reduce employee turnover and save millions of dollars. Other advantages of the Six Sigma process include

- Enhances customer experiences by improving products and reducing customer returns.

- Makes it easier to find problems early and solve them immediately
- Helps you predict the chance of variation in processes, which can improve condition standards, reduce waste and improve profits.
- Eliminates defects, waste and bottlenecks in products.
- Improves stakeholder trust and credibility.

Limitations of SixSigma

- One of the challenges of the fact-driven process of identifying a problem and working toward a solution is that it tends to leave out a key component: humans—and more importantly, how humans impact and work through different obstacles. Sometimes it is often beneficial to give employees a chance to tackle issues head-on before investing in a complete operational overhaul.
- The one-size-fits-all approach to Six Sigma can also be somewhat limiting at times, especially within organizations or disciplines that rely on creativity. Employees who crave the freedom to toss caution (and sometimes process) to the wind in an effort to innovate may find the Six Sigma process stifling.
- SixSigma also does not technically allow for the introduction of new tools or methods, even when they could be beneficial. Since Six Sigma generally requires total dedication across all teams, it’s difficult to use or experiment with other process methodologies for other areas of the organization.