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Unit-III

Acceptance Sampling Plan

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Unit III

Acceptance sampling

Acceptance sampling

In 1920, Harold F. Dodge and Harry G. Roming developed statistical methods for product control known as acceptance sampling or sampling inspection as an alternative of 100% inspection. Now days, product control is achieved through acceptance sampling.

Inspection

- When a manufacturer produces a product or buys some parts of the product from outside agents or suppliers, he/she would like to ensure that the final product is as per specifications. For this purpose, he /she inspect the lot at every strategic point.
- The method of checking, measuring or testing one or more quality characteristics of the product or the parts and determining whether it satisfies the required specifications or not, is called inspection.

Types of Inspection

- Inspection of Variables
    - In this types of inspection, the quality characteristic (s) of an item/ unit is (are) measured and compared with the required specifications.
    - For example, the desired specification for the diameter of a ball bearing is 50mm, length of the refill of a ball pen is 14 cm, weight of a cricket ball is 162 gram, and so on.
- Inspection of Attributes
    - In Inspection of Attributes, actual measurements are not taken. Instead, the item/ unit are categorized as defective or non defective on the basis of Go–No-Go gauges. It means that if a unit fulfils all required quality characteristics, it is categorized as non-defective and if not, it is categorized defective.
    - In such type of inspection, the number of defective units (or defects) is counted.

Methods of inspection

There are two methods of inspection

- **100% inspection**
    - In this method of inspection each and every item /unit of any given lot is inspected. A decision regarding the quality of the entire lot is taken on the basis of all inspected units of the lot.
    - This procedure needs huge expenditure of time, money, lab our and measures. Also, if the product is such that it is completely destroyed under the process of inspection (e.g., a cracker, ammunition, etc.) then in such cases, 100% inspection is neither practicable nor economical. As an alternative, we use sampling inspection.
- **Sampling inspection**
    - In sampling inspection method, some item/ units (called sample) are randomly selected from a lot in such a way that the selected sample is a true representative of the entire lot. Then each and every unit of the selected sample is inspected.
    - A decision regarding the quality of the entire lot is taken on the basis of the information obtained from the sampled units.

Acceptance Sampling Plan

Definition

A sampling inspection in which a decision about acceptance or rejection of a lot is based on one or more samples that have been inspected is known as acceptance sampling.

Advantages of Acceptance Sampling:

The main advantages of acceptance sampling are as follows:

- It is less expensive in terms of money, time and labor in comparison to 100% inspection.
- For items, which cannot be used after single inspection, such as crackers, bulbs, tube lights, food, etc., 100% inspection is not practicable. Sampling inspection is the only way for inspecting such items.
- In acceptance sampling a sample of a small number of items or units is inspected and hence smaller inspection staff is required.
- in many cases, acceptance sampling provides better outgoing quality. In general, it is seen and agreed that good 100% inspection removes only 85% to 90% of the defective items, whereas very good 100% inspection removes only 99% of the defective items. However. Due to human error, it usually does not reach the 100% mark. In other words, we can say that 100% inspection is not always reliable because it involves too much routine work for the persons inspecting each and every item. Due to this, defective items may also be labeled as satisfactory and may accepted at times when these persons are distracted. Hence, an appropriate sampling plan is preferable.
- Due to quick inspection through the acceptance sampling, the scheduling and delivery times are saved.

Limitations of Acceptance Sampling:

- Since in acceptance sampling the entire lot is accepted or rejected on the basis of conclusions drawn from one or more samples, there is always some risk of making wrong inference about the quality of the lot. These risks are termed the producer's risk and consumer's risk.
- The success of acceptance sampling depends on the randomness of the sample, quality characteristics to be tested, lot size, acceptance criteria, etc. Therefore, it is a specialized job requiring careful planning and execution and every one cannot undertake it.

Types of acceptance sampling plan

There are several types of acceptance sampling plans based on different approaches that can influence the decision about a lot.

These are categorized as follows:

i) Acceptance sampling plans for attributes, and

ii) Acceptance sampling plans for variables.

i) Acceptance Sampling Plans for Attributes

Acceptance sampling plans in which actual measurements of the quality characteristics are not made but the units/ items are categorized as defective or non defective on the basis of Go-No-Go gauge are called Acceptance sampling plans for attributes.

These plans are of two types:

- **Lot-by-Lot Acceptance sampling plans for attributes**

Lot-by-Lot Acceptance sampling plans for attributes are the most commonly used sampling plans and therefore are simply called acceptance sampling plans for attributes. Such plans are used whenever the units to be inspected can be conveniently grouped into batches or lots.

In Lot-by-Lot Acceptance sampling plans for attributes, generally, we use the following plans:

- **Single sampling plan**

  In the single sampling plan, the decision about the acceptance or rejection of a lot is based on a single sample that has been inspected. This is the simplest type of sampling plan.

- **Double sampling plan**

  In the double sampling plan the decision about the acceptance or rejection of a lot requires the evidence of two samples drawn from the lot. If the lot quality is good (or bad), the lot is accepted (or rejected) on the basis of the first sample.

  If the first sample shows an intermediate quality, the decision about the lot is taken on the evidence of the first and second sample combined. This is more complicated then the single sampling plan.

- **Multiple sampling plan**

  A multiple sampling plan is an extension of the double sampling plan. This sampling plan may require more than two samples to reach a decision about the acceptance or rejection of a lot.

- **Sequential sampling plan**

  In the sequential sampling plan the sample size is not pre-fixed as in the case of single, double and multiple sampling plans. The units/items are drawn from the lot, one at a time and inspected.

  The decision about accepting or rejecting of the lot or continuing with the inspection by taking one more unit from the lot, is made on the basis of information available up to that stage.

- **Continuous production Acceptance sampling plans for attributes**

Continuous production Acceptance sampling plans for attributes are used when the units/ items to be inspected cannot be grouped into lots or batches. Many manufacturing operations do not crate lots because in these operations, the units are produced in a continuous process on a convey or belt or other straight line systems. Continuous production Acceptance sampling plans for attributes are beyond the scope of this course.

ii) Acceptance Sampling Plans for Variables.

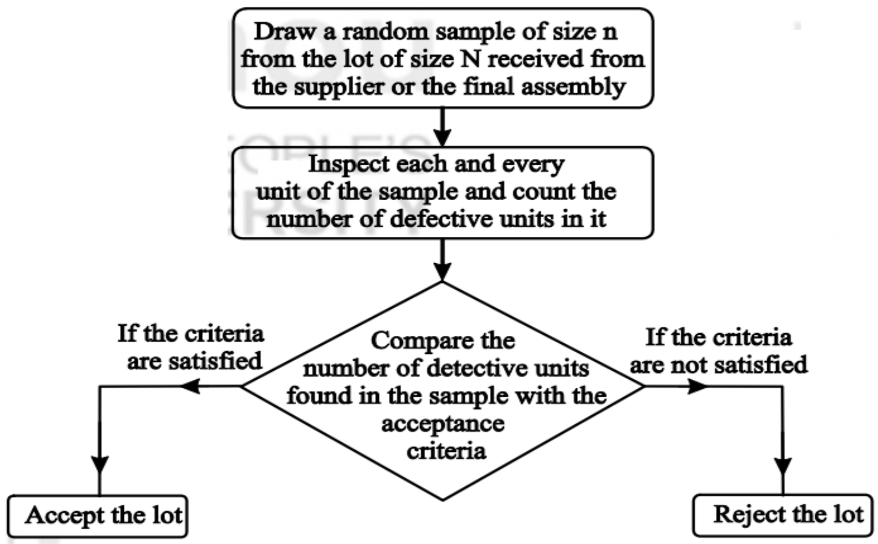
The Acceptance sampling plans in which actual measurements of the quality characteristics are taken are called Acceptance sampling plans for variables.

Implementation of Acceptance Sampling Plans for Attributes

Suppose that lots of the same size, say, N, are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the acceptance sampling plan to arrive at a decision about the lot is described in the following steps:

- Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.

- Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective on the basis of certain criteria. At the end of the inspection, we count the number of defective units found in the sample.
- Step 3: We compare the number of defective units found in the sample with the acceptance criteria.
- Step 4: If the acceptance criteria are satisfied, we accept the entire lot. Otherwise, we reject the entire lot.



Example:

Let us explain these steps further with the help of an example. Suppose a buyer of cricket balls wants to inspect a sample of 20 balls from each lot to check their quality. The acceptance criterion is that if the sample contains at most one defective ball, the lot would be accepted. Otherwise, it would be rejected. The buyer draws 20 balls from each lot and inspects each and every ball of the sample and classifies each ball as defective or non defective on the basis of certain defects. At the end of the inspection, he/she counts the number of defective balls found in the sample and compares the number of defective balls with the acceptance criterion. If the number of defective balls in the sample is more than 1, he/she rejects the lot. If the number of defective balls is zero or 1, he/she accepts the lot.

Terms used in acceptance sampling plans.

❖ **Lot**

- A lot is the collection of units or items from which a sample is taken and inspected to determine its acceptability. It is found that Lot formation influences the effectiveness of the acceptance sampling plan. Therefore, formation of the lot is important for the success of an acceptance sampling plan. Some guidelines for the Lot formation are as follows:
  - Units in the lot should be homogeneous. It means that the lot should consist of units produced by the same machine, same operators, using the same raw materials and approximately during the same time period.

- Units in the lots should be packed so that the shipping costs and handling risks are minimum. This also makes the selection of units in the sample easy
- The decision about the acceptance or rejection of all units of the lot is taken on the basis of the results of inspection of the sample. Therefore, the sample should be such that it is a true representative of the lot.

- **Sentencing**
  - The act of accepting or rejecting the entire lot is called sentencing the lot.
- **Lot Size**
  - The number of units in a lot is called lot size. It is denoted by N.
- **Sample Size**
  - The number of units inspected to sentence a lot is called sample size. It is denoted by n.
- **Lot Quality**
  - The proportion of defective units in a lot is called lot quality or proportion defective. It is denoted by p and defined as
  - P = Number of defective units in a lot / lot size
- **Acceptance Number**
  - When a deal is finalized between a seller and a buyer, they decide on the maximum number of allowable defective units in a sample. This number is called acceptance number and is denoted by c.
  - The rule for acceptance or rejection of a lot is that if the number of defective units observed in the sample is less than or equal to the acceptance number c, the lot will be accepted. Otherwise, it will be rejected.
- **Probability of accepting a lot (Pa)**
  - Suppose that the lots of the same size, say, N, are received from the supplier or the final assembly line and submitted for inspection one at a time.
  - The lot is accepted if the number of defective units observed in the sample is less than acceptance number (c). Otherwise, it is rejected.
  - Suppose a random sample of size n is drawn from each lot for inspection.
- **Acceptance Quality Level (AQL)**
  - The seller and the buyer generally know that the supply of completely defect-free lots is not possible. So they usually negotiate and arrive at an agreement that the buyer will accept all lots which have at most a definite quality level or definite percentage of defective units. This definite quality level is known as acceptance quality level (AQL). It is denoted by p1. Hence, AQL can be defined as follows:
  - Acceptance quality level is the quality level decided in a negotiation of the seller and the buyer: If the proportion of defective units in a lot is less than or equal to AQL, the buyer will have to definitely accept the lot. Otherwise, the buyer may accept or reject the lot.
- **Lot Tolerance Percent Defective (LTPD)**

- In order to reduce the producer's risk, the buyer agrees to tolerate a lot quality worse than acceptance quality level (AQL) up to a certain limit but not beyond it. This limiting value is known as the lot tolerance percent or proportion defective (LTPD). It is also known as reject able quality level (RQL), unsatisfactory quality level or limiting quality level (LQL).
- LTPD is also decided at the time when the acceptance quality level is decided in the negotiation between the producer and the buyer. They make an agreement that the buyer will definitely reject the lot of quality equal to or greater than LTPD. Therefore, LTPD can be defined as follows:
- Lot tolerance percent or proportion defective is the quality level decided in negotiation between the producer and the buyer: If the proportion of defective units in a lot is equal to or greater than this level, the buyer will definitely reject the lot. Otherwise, the buyer may accept or reject the lot.
- The lot tolerance percent defective (LTPD) of a sampling plan is the level of quality at which the lot is routinely rejected by the sampling plan. LTPD is greater than AQL.

Producer's Risk

It may happen in practice that a sampling inspection plan leads to the rejection of a lot of satisfactory or good quality. This means that there is a possibility of rejecting a lot having a quality level less than or equal to the acceptance quality level (AQL) due to sampling inspection. If a lot of good quality is rejected, the producer suffers loss. Therefore, the producer always faces the risk of a good lot being rejected.

Define:

The probability of rejecting a lot of acceptance quality level (AQL) is known as the producer's risk. It is denoted by $P_p(p)$ or in short as P_p and given by

$$P_p(p) = P_p = P[\text{rejecting a lot of acceptance quality level}] = \alpha$$

This can be written as

$$P_p(p) = 1 - P[\text{accepting a lot of acceptance quality level}] = 1 - P_a(p - p_1)$$

The producer's risk is equivalent to the type I error in hypothesis testing MST-004 entitled Statistical Inference. Therefore, it is also denoted by α. For computing the producer's risk, we have to compute the probability of accepting a lot of quality p = AQL.

Consumer's Risk

A person or a firm or an organization that purchases goods for its own need or consumption or for use in the production of other goods (not for resale to another person or firm or organization directly or indirectly) is known as a consumer.

Just as the producer has a risk of a lot of good quality being rejected, a consumer also has a risk of buying a lot of unsatisfactory quality. Such a risk is known as the consumer's risk. If P_2 is the maximum proportion of defective (LTPD) in the lot,

Which the consumer is ready to tolerate, the consumer's risk may be defined as follows: The probability of accepting a lot of unsatisfactory quality, i.e., LTPD is known as consumer's risk. It is denoted by $P_c(p)$ or in short as P_c.

Thus, $P_c(p) = P[\text{accepting a lot of quality } = \text{ LTPD}] = \beta$

Note that $P_c(p)$ is the same as $P_a(p)$ for p = LTPD

Average Outgoing Quality Limit (AOQL)

Average outgoing quality limit defined as the maximum percent of defective items accepted by a given sampling plan assuming that all rejected lots are subjected to 100% inspection and all non conforming items in such lots are replaced with conforming items.

Single sampling plan

A sampling plan in which a decision about the acceptance or rejection of a lot is based on a single sample that has been inspected is known as a single sampling plan.

For example, suppose a buyer purchases cricket balls in lots of 500 from a company manufacturing cricket balls. To check the quality of the lots, the buyer draws a random sample of size 20 from each lot and takes a decision about accepting or rejecting of the lot on the basis of the information provided by this sample. Since the buyer takes the decision about the lot on the basis of a single sample, this sampling plan is a single sampling plan.

A single sampling plan requires the specification of two quantities which are known as parameters of the single sampling plan. These parameters are

n – Size of the sample, and

c – Acceptance number for the sample.

Let us suppose that the lots are of the same size (N) and are submitted for inspection one at a time.

Implementing the Single Sampling Plan

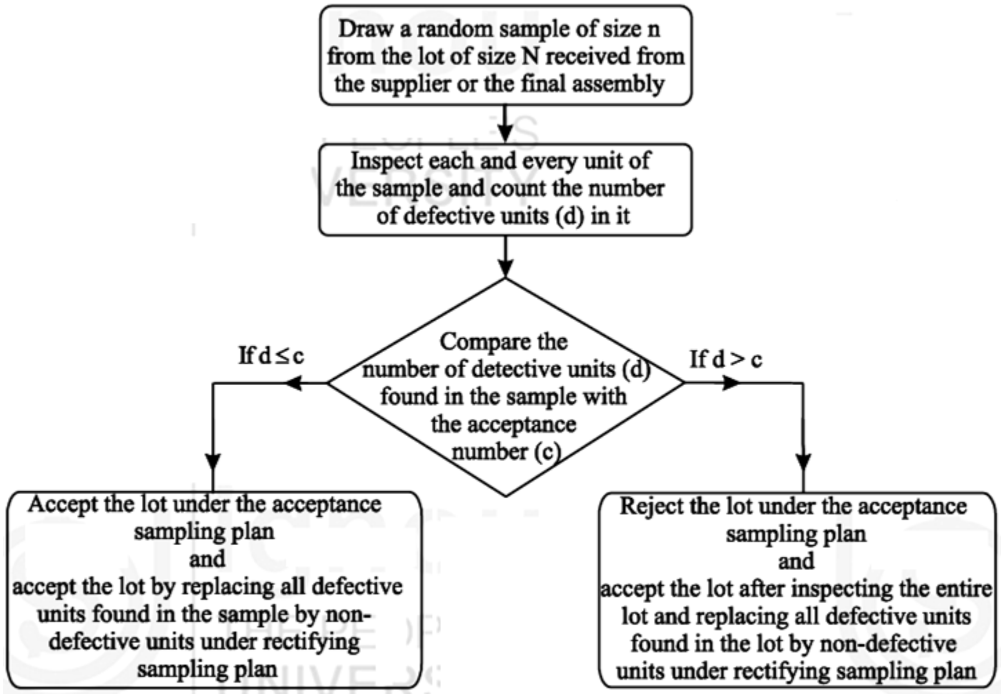
- Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.
- Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective. At the end of the inspection, we count the number of defective units found in the sample. Suppose the number of defective units found in the sample is d.
- Step 3: We compare the number of defective units (d) found in the sample with the stated acceptance number (c).
- Step 4: We take the decision of acceptance or rejection of the lot on the basis of the sample as follows:

  Under acceptance sampling plan:

  If the number of defective units (d) in the sample is less than or equal to the stated acceptance number (c), i.e., if $d \leq c$, we accept the lot and if $d > c$, we reject the lot.

  Under rectifying sampling plan:

  If $d \leq c$, we accept the lot and replace all defective units found in the sample by non-defective units and if $d > c$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units
- The steps described above are shown in Figure



Example 1:
Problem:

Suppose a mobile phone company produces mobiles phones in lots of 100 phones. To check the quality of the lots, the quality inspector of the company uses a single sampling plan with n = 15 and c = 1. Explain the procedure for implementing it.

Solution:

For implementing the single sampling plan, the quality inspector of the company randomly draws a sample of 15 mobile phones from each lot and classifies each mobile of the sample as defective or non-defective.

At the end of the inspection, he/she counts the number of defective mobiles (d) found in the sample and compares it with the acceptance number (c). If d ≤ c (= 1), he/she accepts the lot and if d > c (= 1), he/she rejects the lot under the acceptance sampling plan.

Under rectifying sampling plan, if d ≤ c (= 1), he/she accepts the lot by replacing all defective mobiles found in the sample by non-defective mobiles and if d > c, he/she accepts the lot by inspecting the entire lot and replacing all defective mobiles in the lot by non-defective mobiles.

Operating Characteristic (OC) curve

The operating characteristics (OC) curve is an important aspect of an acceptance sampling plan. This curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected.

In this section, we discuss how to construct the OC curve for a single sampling plan.

That for constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we first compute the probability of accepting a lot of incoming quality p for a single sampling plan.

That the single sample plan, we accept the lot if the number of defective units (d) in the simple is less than or equal to the acceptance number (c) it means that if X represents the number of defective units in the sample,

We accept the lot if X ≤ c, i.e., X=0 or 1 or 2,…or c. therefore, the probability of accepting the lot of incoming quality p is given by

$$P_c(p) = P[X \leq c] = P[X = 0 \; or \; 1, \ldots, or \; c]$$

$$= P[X = 0] + P[X = 1] +, \ldots, + P[X = \; c] (\therefore X = 0,1,2, \ldots., c \; are \; mutually \; exclusive)$$

$$P_c(p) = \sum_{x=0}^{c} P[X = x] \;\; \rightarrow (1)$$

We can calculate this probability if we know the distribution of X. Generally, in quality control, a random sample is drawn from a lot of finite size without replacement. So in such situations, the number of defective units (X) in the sample follows a hyper geometric distribution. In a lot of size N and incoming quality p, the number of defective units is N_p and the number of non defective units is $N - N_p$.

Therefore, the probability of getting exactly X defective units in a sample of size n from this lot is given by

$$P[X = x] = \frac{Np_{C_x} N - Np_{C_{X-x}}}{N_{C_x}} : x = 0,1, \ldots. \min(Np, n) \rightarrow (2)$$

Producer's risk

The probability of rejecting a lot of acceptance quality level (AQL) p_1 is known as the producer's risk. Therefore, the producer's risk for a single sampling plan is given by

$$P_p = P[\text{rejecting a lot of acceptance quality level } \mathrm{p}_1]$$

$$= 1 - P[accep\text{ting a lot of acceptance quality level } \mathrm{p}_1]$$

$$P_p = 1 - P_a(\mathrm{p}_1)$$

$$P_c(p) = P[X \leq c] = P[X = 0 \; or \; 1, \ldots, or \; c]$$

$$= P[X = 0] + P[X = 1] +, \ldots, + P[X = \; c] (\therefore X = 0,1,2, \ldots., c \; are \; mutually \; exclusive)$$

$$P_c(p) = \sum_{x=0}^{c} P[X = x] \;\; \rightarrow (1)$$

We can calculate this probability if we know the distribution of X. Generally, in quality control, a random sample is drawn from a lot of finite size without replacement. So in such situations, the number of defective units (X) in the sample follows a hyper geometric distribution. In a lot of size N and incoming quality p, the number of defective units is Np and the number of non defective units is N-Np. Therefore, the probability of getting exactly X defective units in a sample of size n from this lot is given by

$$P[X = x] = \frac{Np_{C_x} N - Np_{C_{X-x}}}{N_{C_x}} : x = 0,1, \dots . \min(Np, n) \rightarrow (2)$$

Therefore, the producer's risk for a single sampling plan is given by

$$P_p = P[\text{rejecting a lot of acceptance quality level } \mathrm{p}_1]$$
$$= 1 - P[\text{accepting a lot of acceptance quality level } \mathrm{p}_1]$$
$$= 1 - P_a(\mathrm{p}_1)$$

We can compute $P_a(\mathrm{p}_1)$ from the quality level p with (p_1) as follows

$$P_a(\mathrm{p}_1) = \sum_{x=0}^{n} P[X = x] = \frac{Np_{C_x} N - Np_{C_{n-x}}}{N_{C_x}}$$

Therefore the producer's risk is given by

$$\mathrm{P_p} = 1 - P_a(\mathrm{p}_1) = 1 - \sum_{x=0}^{n} \frac{Np_{C_x} N - Np_{C_{n-x}}}{N_{C_x}}$$

Consumer's Risk

The Consumer's Risk for a single sampling plan: By definition, the probability of accepting a lot of unsatisfactory quality (LTPD) p_2 is known as the consumer's risk. Therefore, the consumer's risk for a single sampling plan is given by

$$P_c = P[\text{accepting a lot of quality level } \mathrm{p}_2]$$

Then compute the consumer's risk for the single sampling plan from the quality level p with p_2 as follows:

$$P_c = P_a(\mathrm{p}_2) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{Np_{C_x} N - Np_{C_{n-x}}}{N_{C_x}}$$

Average Outgoing Quality (AOQ)

The concept of average outgoing quality is particularly useful in the rectifying sampling plan where the rejected lots are inspected 100% and all defective units are replaced by non-defective units.

Define:

The expected quality of the lots after the applications of sampling inspection is called the average outgoing quality (AOQ). It is calculated from the formula given below:

$$\text{AOQ} = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$$

So in a single sample plan, we can obtain the formula for average outgoing quality by considering the following situations:

- ❖ If the lot of size N is accepted on the basis of a sample of size n, (N – n) units remain un-inspected. If the incoming quality of the lot is p, we expect that p (N – n) defective units are left in the lot after the inspection.
- ❖ However, the probability that the lot will be accepted by the sampling plan is Pa. Therefore, the expected number of defective units per lot in the outgoing stage is p (N – n) Pa.

$$p(N - n)P_a + 0 = p(N - n)P_a.$$

Hence, the average proportion defective in the outgoing stage or average outgoing quality (AOQ) is given by

$$\text{AOQ} = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$$

$$\text{AOQ} = \frac{p(N - n)P_a}{N}.$$

If the sample size n is very small in proportion to the lot size N, i.e., n / N, for AOQ becomes

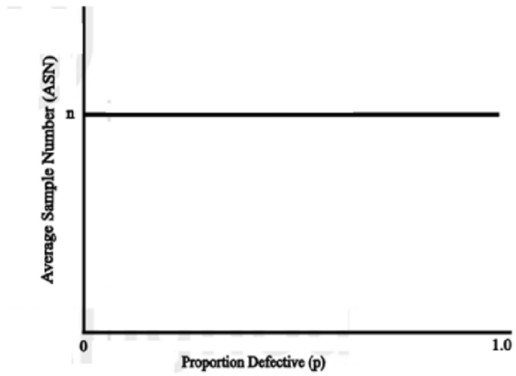
$$\text{AOQ} = p\left(1 - \frac{n}{N}\right)P_a(N - n)pP_a$$

Let us now discuss the construction of the AOQ curve for a single sampling. As you know, the AOQ curve is constructed by taking the quality level (proportion defective) on the X-axis and the AOQ on the Y-axis. So for constructing the AOQ curve, we first consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding AOQ. Let us consider some examples for calculating AOQ and constructing the AOQ curve.

Average Sample Number (ASN):

The average sample number is the expected Single Sampling Plans number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan. In acceptance single sampling plan, the decision about the acceptance or rejection of a lot is taken on the basis of a single sample that has been inspected. Therefore, the ASN in a single sampling plan is simply the sample size n. It means that ASN is constant in a single sampling plan. Therefore, ASN = n

The curve drawn between the ASN and the lot quality (p) is known as the ASN curve.
The ASN curve for a single sampling plan is a straight line as shown in



The quality control inspector takes the decision of acceptance or rejection of the lot on the basis of the single sample of size n = 15. So the ASN for this single sampling plan is 15.

Average Total Inspection (ATI):

The concept of average total inspection (ATI) is considered under rectifying sampling plan in which rejected lots undergo 100% inspection. It is defined as follows: The average number of units inspected per lot under the rectifying sampling plan is called the average total inspection (ATI). So in a rectifying single sampling plan, the number of units to be inspected will depend on two situations given below:

- If the lot of size N is accepted on the basis of a sample of size n, the number of units inspected is n and the probability of the accepting the lot is Pa.
- If the lot is rejected on the basis of a sample, we inspect the entire lot of size N and the probability of rejecting the lot is (1 – Pa).

Therefore, we can compute the ATI for a single sampling plan as follows:
ATI = Average number of units inspected per lot.

Double sampling plan

A sampling plan in which a decision about the acceptance or rejection of a lot is based on two samples that have been inspected is known as a double sampling plan. The double sampling plan is used when a clear decision about acceptance or rejection of a lot cannot be taken on the basis of a single sample.

In double sampling plan, generally, the decision of acceptance or rejection of a lot is taken on the basis of two samples. If the first sample is bad, the lot may be rejected on the first sample and a second sample need not be drawn. If the first sample is good, the lot may be accepted on the first sample and a second sample is not needed. But if the first sample is neither good nor bad and there is a doubt about its results, we take a second sample and the decision of acceptance or rejection of a lot is taken on the basis of the evidence obtained from both the first and the second samples.

Since the decision of acceptance or rejection of the lot is taken on the basis of two samples, this is a double sampling plan. A double sampling plan requires the specification of four quantities which are known as its parameters.

These parameters are

- n_1 – size of the first sample,
- c_1 – acceptance number for the first sample,
- n_2 – size of the second sample, and
- c_2– Acceptance numbers for both samples combined.

So far you have learnt the definition of the double sampling plan and why it is used. We now describe the procedure for implementing it and its advantages over the single sampling plan.

Implementation of Double Sampling Plan

Suppose, lots of the same size, say N, are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the double sampling plan to arrive at a decision about the lot is described in the following steps:

- Step 1: We draw a random sample (first sample) of size n1 from the lot received from the supplier or the final assembly.
- Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective. At the end of the inspection, we count the number of defective units found in the sample. Suppose the number of defective units found in the first sample is d_1.
- Step 3: We compare the number of defective units (d_1) found in the first sample with the stated acceptance numbers c_1 and c_2.
- Step 4: We take the decision on the basis of the first sample as follows: Under acceptance sampling plan If the number of defective units (d_1) in the first sample is less than or equal to the stated acceptance number (c_1) for the first sample, i.e., if
  - $\mathrm{d}_1 \leq \mathrm{c}_1$, we accept the lot and if
  - $\mathrm{d}_1 \leq \mathrm{c}_{2,}$ we reject the lot. but if
  - $\mathrm{c}_1 < \mathrm{d}_1 \leq \mathrm{d}_1 \leq \mathrm{c}_{2,}$ the first (single) sample is failed.

  Under rectifying sampling plan

  - If $\mathrm{d}_1 < \mathrm{c}_1$, we accept the lot and replace all defective units found in the sample by non-defective units.
  - If $\mathrm{d}_1 > \mathrm{c}_{2,}$ We accept the lot after inspecting the entire lot and replacing all defective units in the lot by non defective units. But if $\mathrm{c}_1 < \mathrm{d}_1 < \mathrm{c}_2$, the first (single) sample is failed.
- Step 5: If $\mathrm{c}_1 < \mathrm{d}_1 < \mathrm{c}_2$ we draw a second random sample of size n_2, from the lot

❖ Step 6: We inspect each and every unit of the second sample and count the number of defective units found in it. Suppose the number of defective units found in the second sample is d_2.

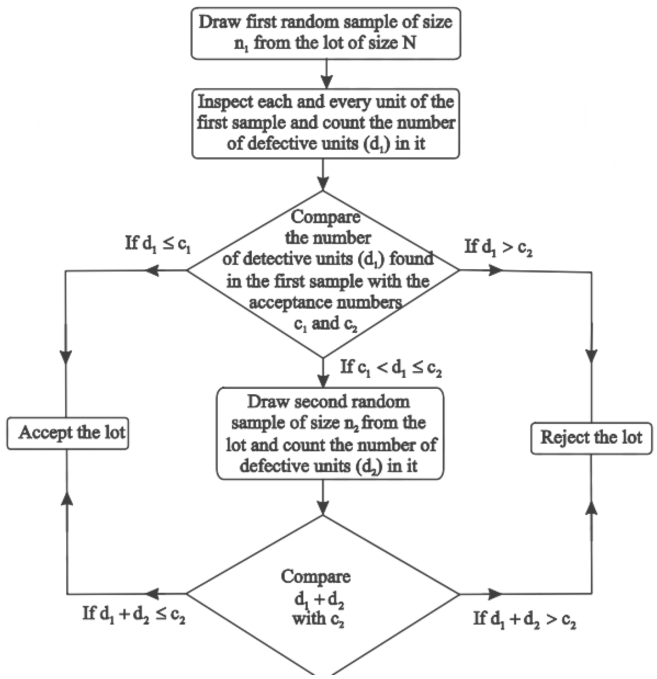
❖ Step 7: We combine the number of defective units (d_1 and d_2) found in both samples and consider ($d_1 + d_2$) for taking the decision about the lot on the basis of the second sample as follows:

Under acceptance sampling plan

- If $d_1 + d_2 \leq c_2$, we accept the lot and if $d_1 + d_2 > c_2$. We reject the lot.

Under rectifying sampling plan

- If $d_1 + d_2 \leq c_2$, we accept the lot and replace all defective units found In the second sample by non-defective units.
- If $d_1 + d_2 > c_2$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units. The steps described above are shown in figure for the acceptance sampling plan.



Advantages of Double Sampling Plan

A double sampling plan has the following two main advantages over a single sampling plan:

- The principal advantage of the double sampling plan over the single sample plan is that for the same degree of protection (i.e., the same probability of accepting a lot of a given quality), the double sampling plan may have a smaller average sample number (ASN) than that corresponding to the single sampling plan. The underlying reason is that the size (n1) of the first sample in the double sampling plan is always smaller than the sample size (n) of an equivalent single sampling plan. Thus, if a decision is taken on the basis of the first sample, ASN will be lower for the double sampling plan or if a decision is taken after the second sample, the ASN will be reduced.

- The double sampling plan has the psychological advantage of giving a lot a second chance. From the viewpoint of the producer/manufacture, it is unfair to reject a lot on the basis of a single sample. The double sampling plan permits the decision to be made on the basis of two samples. However, double sampling plans are costlier to administer in comparison with the single sampling plans. Try the following exercise to explain the procedure for implementing a double sampling plan.

Difference between Single and Double Sampling Plans

The differences between single and double sampling plans are given below:

- Single sampling plans are easier to design and operate in comparison with the double sampling plans.
- The advantage of double sampling plans over single sampling plans seems to be psychological. It looks unreasonable to reject a lot on the basis of one sample and it is more convincing to say that the lot was rejected on the basis of inspection of two samples.
- Under the double sampling plan, the good lot will generally be accepted and the bad lot will usually be rejected on the basis of the first sample. Thus, in all cases, where a decision to accept or reject is taken on the basis of the first sample, there is a considerable saving in the amount of inspection than required by a comparable single sampling plan. Moreover, when a second sample is taken, it may be possible to reject the lot without completely inspecting the entire second sample.
- In the double sampling plan, on an average, 25% to 33 % less number of items/units need to be inspected as compared to the single sampling plan.
- The operating characteristic (OC) curve for a double sampling plan is steeper than a single sampling plan, i.e., the discriminatory power of the double sampling plan is higher than that of the single sampling plan. So far you have learnt what a double sampling plan is and how it is implemented in industry.

Operating Characteristic (OC) Curve

The operating characteristic (OC) curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will either be accepted or rejected.

Product Control

Constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we now describe how to compute the probability of accepting a lot in a double sampling plan. You have learnt in that in a double sampling plan, the decision of acceptance or rejection of the lot is taken on the basis of two samples. The lot is accepted on the first sample if the number of defective units (d_1) in the first sample is less than the acceptance number c_1. The lot is accepted on the second sample if the number of defective units (d_1 + d_2) in both samples is greater than $c_1$and less than or equal to the acceptance number c_2.

Therefore, if $Pa_1(p)$ and Pa_2 (p) denote the probabilities of accepting a lot on the first sample and the second sample, respectively, the probability of accepting a lot of quality level p is given by:

$$P_a(p) = Pa_1(p) + Pa_1(p)$$

If X and Y denote the observed number of defective units in the first and second samples, respectively, we accept the lot on the first sample if $X \leq c_1$, i.e., if X=0 or 1 or 2……or c_1

Therefore,

$$P_{a1}(p) = P[X \leq c_1] = P[X = 0 \text{ or } 1 \text{ or } 2 \ldots \ldots \text{ or } c_1$$

$$= P[X = 0] + P[X = 1] + \cdots + P[X = c_1] \ (\because \ X = 0,1,2 \ldots, c_1 \text{ are mutually exclusive})$$

$$= \sum_{X=0}^{c_1} P[X = x]$$

Curtailing of Inspection

Curtailing of inspection is when the inspection is stopped short of the required sample size because the accept/reject decision has been determined regardless of the results of the remaining units.

For example, suppose the single sampling plan for defective units n=50 and a=1 is being used. If following the inspection of just 10 units, 2 defectives are found, regardless of the results of the remaining 40 units, the lot will be rejected.

Inspection can be curtailed when inspecting for defective units, defects per unit, and defects per quantity. Curtailing can occur on both acceptance and rejection and on each stage of a double or multiple sampling plan. Curtailing does not affect the protection provided by a sampling plan including the OC curve, AOQ curve, AQL, LTPD, and AOQL.

It does reduce the ASN curve. However, despite this benefit, curtailing of inspection is rarely performed because of the added complexity and because it interferes with other uses of the data such as trending.

Military standard 105E

- Sampling procedure for inspection by attributes developed during world war II and is the most widely used acceptance sampling system for attributes in the world today
- A collection of sampling schemes; therefore an acceptance sampling system
- Provides for three types of sampling; single double and multiple
- Primary focal point is the acceptable quality level(AQL)
- Different AQLs may be designated for different types of defects; critical, major, and minor.
- Generally specified in contract or by authority responsible for sampling
- Sample size is determined by lot size and by choice of inspection level.

Description of the standard

Standard sampling procedures for Inspection by attributes were developed during world war ii MILS20 105E is the most widely used acceptance sampling system for attributes in the world today The original version of the standard, MALSTD 105A was issued in 1950: since then there is your perisions The last version, M21 STD1055, was issued in 1989.

The sampling plans discussed in previous section. sf the Eapter are Individual sampling plans A Sampling scheme is an overall strategy specifying the way in which sampling plans are to be used.MIL STD 105E is a collection of sampling schemes, therefore its an acceptance Sampling, system: our discussion w focus primarily on MILSTD 105 E, however, there is denvative civilian standard, ANSI/ASQC 214, which is quite similar to the military Standard. The standard was also adopted by the International organization for Standardization as ISO, 2859

The standard provides for three types of sampling

- Single sampling
- Double sampling
- multiple sampling

For each type of sampling plan, a provision is mode for either normal Inspection, tightened Inspection, or reduced Inspection - Normal Inspection is used at the inspection activity Tightened Inspection Instituted when the supplier's relent quality history has deteriorate. Acceptance requirements for 106 using tightened inspection are more stringent then under normal inspection reduced Inspection & instituted Inspection when the supplier’s silent quality history tas been exceptionally good it he sample size generally word under reduced Inspection is less than that under normal Inspection

The primary focal point of MILSTD 105E the acceptance Quality Level (AQI). The standard is Indicated with of respect to a series of AQL's when the wipers standard is used for percent defective plans, the AQL's range from 0.10% 50 10% 30% defect per units plan, there are an additional ten as running upto 1000 defects per 100 unit. at should be noted that for the Smaller

AAL levels, the same sampling plan can be used to control efther a fraction defective ora nog defect per unit. The AQL's are arranged in a progression, each AQLusing approximately 1.585 times the proceeding one.

1. Normal to tightened.

When normal inspection is in effect Heightened inspection is instituted when two out of five consecutive lot's have been rejected on original submission.

2. tighted to normal

When tighted inspection is in effect, normal inspection is instituted when for consecutive lots or batches are accepted on original inspection.

3. Normal to reduced

When normal inspection to in effects reduced inspection is instituted provides all four of the following conditions are satisfied.

- The preceding tim lots have been on normal inspection and none of the lots has been rejected on original inspection.
- The total no of defectives is the samples from the proceeding ten lots is less than or equal to the applicable limit no specified in the standard.
- Production is at a steady rate that is, no difficulty such as machine breakdowns, materials Shortage or other problems have recently occurred.
- Reduced Inspection is considered described by the authority responsible for population.

4. Reduced to Normal

When reduced Inspection is in effects normal Inspection is Instituted when any of the following your conditions. A lot or catch is rejected

When the sampling procedure terminates with neither acceptance, nor rejection criteria having been met, the tot or betch will be accepted, but normal Inspection is reinstituted starting with the next lot