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## Tiruchirappalli- 620024

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# Unit-IV

# Inventory Theory

**Dr. T. Jai Sankar**
**Associate Professor and Head**
**Department of Statistics**

**Ms. E. Devi**
**Guest Faculty**
**Department of Statistics**

# 8

## INVENTORY MODELS

**Structure**



### 8.1. INTRODUCTION

In this chapter, we shall introduce the concept of inventory and discuss different kinds of inventory models. Inventory may be defined as a stock of idle resources of any kind having an economic value kept for the purpose of future affairs. The term is generally used to indicate physical resources such as raw materials, semi-finished goods used in the production process, finished products ready for delivery to consumers, human resources such as unused labour or financial resources such as operating capital etc. stock or order to meet an expected demand in the future.

In the present chapter, we discuss the models for inventory control known as economic order quantity (EOQ) models. EOQ models help in deciding as to how much quantity should be kept in stock so that one can balance the costs of holding too much stock. Here, we discuss inventory control, various factors influencing inventory and describe models for determining the EOQ: (i)when demand is uniform; (ii) when shortages are allowed;

(iii) when replenishment is uniform; (iv) when price breaks.

**8.1.1. Objectives:** The objective of these contents is to get familiar reader with the concept of inventory. After studying this chapter, reader should be able to describe the following concepts like:

(i) Inventory Control

(ii) EOQ Model with Uniform Demand

(iii) EOQ Model with uniform replenishment

(iv) EOQ with different rates of demands in different cycles

(v) EOQ Model when shortages are allowed

(vi) EOQ Model with price breaks

## 8.2. MEANING OF INVENTORY CONTROL

Inventories are essential for business and maintenance of inventories costs money by way of expenses on stores, personnel, equipment, insurance etc. Thus, excess inventories are undesirable. Therefore, controlling the inventories in the most profitable way is the need of every business.

### 8.2.1. Necessity for Maintaining Inventory

Though inventory of materials is an idle sequence (since the materials lie idle and are not to be used immediately) almost every organisation must maintain it for efficient and smooth running of its operations, if an enterprise has no inventory of materials at all. On receiving a sales order, it will have to place order for purchase of raw materials, wait for their receipt and then start production. The customer will thus have to wait for a long time for the delivery of the goods and may turn to other suppliers resulting in loss of business for the enterprise. Maintain an inventory is necessary because of the following reasons:

1. It helps in smooth and efficient running of an enterprise.
2. It provides service to the customer at a short notice timely delivery can fetch more goodwill and order.
3. In the absence of inventory, the enterprise may have to pay high prices because of piece-wise purchasing. Maintaining of inventory may earn price discount because of bulk purchasing.
4. It reduces product cost because there is an additional advantage of batching and long production runs.
5. It acts as a buffer stock when raw materials are received late and so many sale orders are likely to be rejected.

6. Process and movement inventories (also called pipe line stocks) are quite necessary in big enterprises where significant amounts of times are required to tranship items from one location to another.
7. Bulk purchases will entail less orders and therefore fewer clerical costs.
8. It helps in maintaining economy by absorbing some of the fluctuations when the demand for an item fluctuates and is seasonal.

### 8.2.2. Factors Influencing Inventories

The major problem of inventory control is to answer two questions:

1. How much to order?
2. When to order?

These are answered by developing a model. An inventory model is based on the consideration of the main aspects of inventory. The varieties of factors related to these are placed below:

**1. Inventory related costs**

Various costs associated with inventory control are often classified as follows:

**(a) Purchase (or production) cost**. The cost of purchasing (or producing) a unit of an item is known as purchase (or production) cost. The purchase price will become important when quantity discounts are allowed for purchases above a certain quantity.

**(b) Ordering (or set up) cost**. If any item is purchased, an ordering cost is incurred each time an order is placed. This cost includes the following factors: administrative (paper work, telephone calls, postage), transportation of items ordered, receiving and inspection of goods etc. If a firm produces its own inventory instead of purchasing the same from an outside source, then production set-up costs are analogous to ordering costs.

**(c) Carrying (or holding) cost**. Holding cost represents the cost of maintaining inventory in stock. It includes the interest on capital, sent for space used for storage, insurance of stored equipment, depreciation, taxes, etc.

**(d) Shortage (or stock out) cost**. The penalty cost for running out of stock (i.e. when an item cannot be supplied on the customer's demand) is known as shortage cost. This cost includes the loss of potential profit through the sale of items demanded and loss of goodwill, in terms of permanent loss of customers and its associated lost profit in future sales.

**(e) Salvage cost (or selling price)**. When the demand for an item is affected by its quantity in stock, the decision depends upon the underlying criterion and includes the revenue from sale of the item. Salvage cost is generally combined with the storage cost and hence is neglected.

**2. Demand**: Demand is the number of units required per period and may be either known exactly or known in terms of probabilities. Problems in which demand is known with certainty, are called deterministic problems, whereas problems in which demand is assumed to be a random variable are called **probabilistic problems**.

**3. Order cycle**. The time period between placement of two successive orders is referred to as an order cycle. The order may be placed on the basis of the following two types of inventory review systems.

**(a) Continuous review**. The record of the inventory level is checked continuously until a specified point (called reorder point) is reached when a new order is placed.

**(b) Periodic review**. In this system, the inventory levels are reviewed at equal time intervals and the orders are placed at such levels.

**4. Time horizon**. The time period over which the inventory level will be controlled is referred to as time horizon. This can be finite or infinite depending upon the nature of demand. This is also known as the planning period over which the inventory is to be controlled. Mostly inventory planning in an enterprise is done on annual basis.

**5. Lead time**. The time between placing an order and its arrival in stock is known as lead time. The lead time can be either deterministic or probabilistic. If the lead time is zero, there is no need for placing an order in advance. If the lead time is known and is not equal to zero and also the demand is known, then it is required to place an order in advance-by-an amount of time equal to the lead time.

**6. Re-order level**. The level between maximum and minimum stock, at which the purchasing (or manufacturing) activities must start for replenishment is known as reorder level.

**7. Stock replenishment**: The rate at which items are added to inventory is one of the important parameters in inventory models. The actual replenishment of items may occur instantaneously or gradually. Instantaneous replacement is possible when the stock is purchased from outside sources while gradual replenishment is possible when the product is manufactured by the company.

**8. Re-order quantity**: This is the quantity of replenishment order. In certain cases, it is the Economic Order Quantity.

**Some notations used in the models**

The notations used in the development of models are as follows:

$Q$ = number of units ordered per order

$D$ = demand in units of inventory pr time period

$N$ = number of orders placed per time period

$TC$ = total inventory cost

$C_0$ = ordering cost (or setup cost per production run) per order

$C_p$ = Carrying or holding cost per unit per period of time the inventory is held

$C$ = Purchase or manufacturing price per unit inventory

$C_s$ = shortage cost per unit of inventory

$L$ = lead time

$T$ = reorder cycle time i.e. time period between placement of two successive orders as a fractional part of standard time horizon

$r_p$ = replenishment rate at which lot size $Q$ is added to the inventory

## 8.3. ECONOMIC ORDER QUANTITY (EOQ)

'Economic Order' Quantity or 'Economic lot size' was first developed by Ford W Harris in 1913 in order to balance costs of holding two much stock against that of ordering in small quantities too frequently.

Economic order quantity is the size of the order representing standard quality of material and is the one for which the aggregate of the costs of procuring the inventory and costs of holding the inventory is minimum.

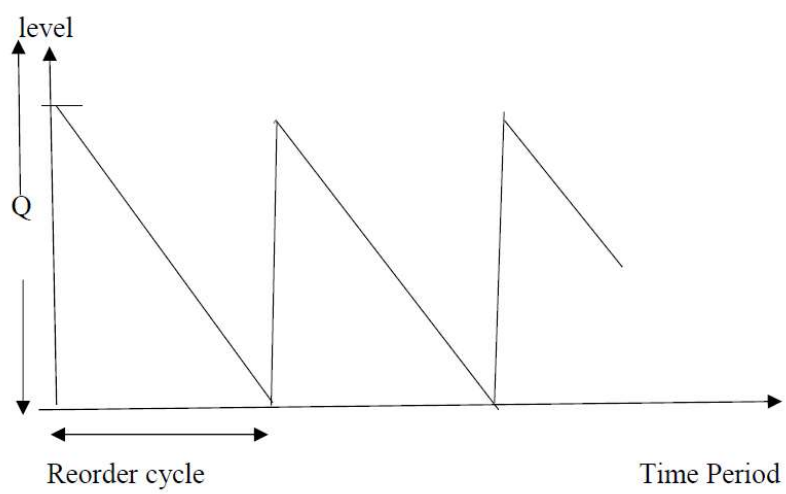
### 8.3.1. Model I: EOQ Model with Uniform Demand

The objective of the model is to determine an optimum EOQ such that the total inventory cost is minimum. Following assumptions are made for this model:

1. Demand $D$ is constant and known.
2. Replenishment is instantaneous i.e. the entire order quantity $Q$ is received at one time as soon as the order is released.
3. Lead time is zero.
4. Purchase price or cost per unit is constant i.e. discounts are not allowed.

   Carry cost $(C_h)$ and ordering cost $(C_o)$ are known and constant.
6. Shortage is not allowed.

Inventory




Since lead time is zero and replenishment is instantaneous safely stock is not required i.e. minimum level is zero. Also, demand is uniform. So, the average inventory per cycle

$= \frac{1}{2}$ (maximum level + minimum level) $= \frac{1}{2}(Q+O) = \frac{Q}{2}$

Since average inventory during any cycle period is $\frac{Q}{2}$, the average inventory during the entire period is also $\frac{Q}{2}$.

So, carrying cost = average units in inventory × carrying cost per unit $= \frac{Q}{2} C_h$

Ordering cost = number of orders × ordering cost per order

$$= N \times C_o$$

$$= \frac{D}{Q} C_o$$

Total variable inventory cost is then given by

$$TC = \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

Differentiating w.r.t. $Q$, we get

$$\frac{d}{dQ}(TC) = -\frac{DC_o}{Q^2} + \frac{C_h}{2}$$

$$\frac{d^2}{dQ^2}(TC) = \frac{2DC_o}{Q^3}$$

For maxima or minima,

$$\frac{d}{dQ}(TC) = 0$$

i.e. $-\frac{DC_o}{Q^2} + \frac{C_h}{2} = 0$

i.e. $\frac{C_o D}{Q} = \frac{C_h Q}{2}$

i.e. $Q = \sqrt{\frac{2DC_o}{C_h}}$

At $Q = \sqrt{\frac{2DC_o}{C_h}}, \frac{d^2}{dQ^2}(TC) > 0$

Hence $TC$ is minimum when $Q = \sqrt{\frac{2DC_o}{C_h}}$.

$\therefore$ EOQ is given as $Q^* = \sqrt{\frac{2DC_0}{C_h}}$

Optimum number of orders placed per time period $\left(N^*\right) = \frac{D}{Q^*}$

Minimum total variable inventory cost

$$= \frac{D}{Q^*}C_o + \frac{Q^*}{r^2}C_h = \sqrt{2DC_oC_h}$$

$$= \sqrt{2 \times \text{demand rate } \times \text{ ordering cost } \times \text{ holding cost}}$$

Optimum length of time between orders $= \frac{Q^*}{D}$

$$= \frac{1}{D}\sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2C_o}{DC_h}}$$

**Note**: If the carrying cost is given as a percentage of average value of inventory held, then total annual carrying cost $C_h$ may be expressed as

$C_h =$ cost of one unit × inventory carrying cost in percentage = $CI$

Hence optimum order quantity is given by

$$Q^* = \sqrt{\frac{2DC_o}{CI}}$$

**8.3.1.1. Example**. The demand rate of a particular item is 12000 units per year. The set-up cost per run is Rs. 350 and the holding cost is Rs. 20 per unit per month. If no shortages are allowed and the replacement is instantaneous, determine

(i) the optimum run size,

(ii) the optimum scheduling period, and

(iii) minimum total expected annual cost.

**Solution.** Here, $D$ = 12000 per year

$C_0$ = Rs. 350

$C_h$ = Rs. 0.2 per unit per month

= Rs. 2.4 per unit per year

(i) Optimum lot size $= Q^* = \sqrt{\dfrac{2DC_o}{C_h}} = \sqrt{\dfrac{2\times 12000\times 350}{2.4}}$

$= \sqrt{3500000} = 1870.8 \simeq 1871$ units

(ii) Optimum scheduling period

$= t^* = \dfrac{Q^*}{D} = \dfrac{1871}{12000}$ years $= 1.87$ months

(iii) minimum total expected annual cost $= \sqrt{2DC_oC_h}$

$= \sqrt{2\times 12000\times 350\times 2.4}$

$= \sqrt{20160000} = \text{Rs.}\ 4490$

**8.3.1.2. Example**. The annual requirement for a product is 3000 units. The ordering cost is Rs. 100 per order. The cost per unit is Rs. 10. The carrying cost per unit per year is 50% of

the unit cost. Find the EOQ. If a new EOQ is found by using ordering cost as Rs. 80, what would be the further savings in cost?

**Solution.** Here $D$ = 300 units per year

$C_o$ = Rs. 100

$C$ = Rs. 10, $I$ = 30%

$C_h = CI = 10\times\dfrac{30}{100}$ = Rs. 3 per unit per year.

Optimal lot size, $Q^* = \sqrt{\dfrac{2DC_o}{C_h}} = \sqrt{\dfrac{2\times 3000\times 100}{3}}$ = 447 units

Total inventory cost $= \sqrt{2DC_0C_h}$

$= \sqrt{2\times 3000\times 100\times 3}$ = Rs. 1342 per year

In the second part, we have $D$ = 3000, $C_o$ = Rs. 100/80, $C_h$ = Rs. 3 per unit per year

$Q^* = \sqrt{\dfrac{2DC_0}{C_h}} = \sqrt{\dfrac{2\times 3000\times 80}{2}}$ = 400 units.

Total inventory cost $= \sqrt{2DC_oC_h} = \sqrt{2\times 3000\times 80\times 3}$ = Rs. 1200 per year

Net savings = 1342 – 1200 = Rs. 142 per year.

**8.3.1.3. Example**. A company requires 1000 units per month. Order cost is estimated to be Rs. 50 per order. In addition to Re 1.00, the carrying costs are 10% per unit of average inventory per year. The purchase price is Rs. 10 per unit. Find the economic lot size to be ordered and the total minimum cost.

**Solution.** Here $D$ = 1000 units per month

= 12000 units per year

$C_0$ = Rs. 50 per order

$C$ = Rs. 10 per unit

$C_h$ = 1.00 + 10% of Rs. 10

= Rs. 2 per unit of average inventory

The economic lot size is given by

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2\times 12000\times 50}{2}} = 775 \text{ units}$$

Total minimum cost = Total minimum inventory cost + Cost of material

$$= \sqrt{2DC_oC_h} + 12000\times 10$$

$$= \sqrt{2\times 12000\times 50\times 2} + 1,20,000$$

= 1549 + 1,20,000 = Rs. 1,21,549.

**8.3.1.4. Example**. The XYZ manufacturing company has determined from an analysis of its accounting and production data for 'part number alpha', that its cost to purchase is Rs. 36 per order and Rs. 2 per part. Its inventory carrying charge is 9 per cent of the average inventory. The demand of this part is 10,000 units per annum. Determine

(i) What should be the economic order quantity?

(ii) What is the optimum number of orders?

(iii) What is the optimum number of days supply per optimum order?

**Solution.** Here, demand per annum, $D$ = 10,000

Ordering cost per order, $C_o$ = Rs. 36

Cost of one part, $C$ = Rs. 2, $I$ = 9%

Inventory carrying cost, $C_h$ = Re 0.09 × 2 = Rs. 0.18

Total inventory cost, $TC$ = Ordering cost + Carrying cost

$$=\left(\frac{10000}{Q}\right)36+0.09\,Q$$

Differentiate w.r.t. $Q$,

$$\frac{d(TC)}{dQ}=-\frac{360000}{Q^2}+0.09$$

Equating it to zero, we have $-\frac{360000}{Q^2}+0.09=0$

$$\therefore Q^2=\frac{360000}{0.09}=4000000$$

$$\Rightarrow Q=2000$$

$$\frac{d^2(TC)}{dQ^2}=\frac{720000}{Q^3}$$

At $Q = 2000$, $\frac{d^2(TC)}{dQ^2}=\frac{720000}{(2000)^3}>0$

$\therefore TC$ is minimum when $Q = 2000$ units

Thus, EOQ is $Q^* = 2000$ units.

(ii) Optimal number of orders = $\frac{\text{Demand}}{\text{EOQ}}=\frac{10000}{2000}=5$

(iii) Optimal number of days supply per optimum order $=\frac{365}{5}=73$ days

**8.3.1.5. Exercises.**

**1.** A company uses annually 12000 units of raw material costing Rs. 1.25 per unit. Placing each order costs Rs. 15 and the carrying costs are 15% per year per unit of average inventory. Find the economic order quantity?

**Answer.** $Q^*=\sqrt{\frac{2DC_o}{C_h}}=\sqrt{\frac{2\times12000\times15}{0.15\times1.25}}$ = 1385 units.

**2.** A manufacturer uses Rs. 10,000 worth of an item during the year. He has estimated the ordering costs is Rs. 25 per order and carrying costs as 12.5% of average inventory value. Find the optimal order size, number of orders per year, time period per order and total cost.

**Answer.** $Q^*=\sqrt{\frac{2DC_o}{C_h}}=\sqrt{\frac{2\times10000\times25}{0.125}}$ = Rs. 2000

Number of orders per year $=\frac{D}{Q^*}=5$

Time period per order $= \frac{365}{5} = 73$ days

Total cost, $TC^* = \sqrt{2 \cdot DC_o C_h} = \sqrt{2 \times 10000 \times 25 \times 0.125}$ = Rs. 250 (variable cost)

Total annual cost = Rs. 10000 + Rs. 250= Rs. 10,250

### 8.3.2. Model II: EOQ with Finite Rate of Replenishment or EOQ with Uniform Replenishment

For this model, it is assumed that the production run may take a significant time to complete. Let $r_d$ be the demand rate in units per unit of time and $r_p$ be the replenishment rate per unit time. Assume that each cycle time *t* of two parts $t_1$ and $t_2$ such that

(a) production is continuous and constant until $Q$ units are produced to stock, then it stops;

(b) the production rate $r_p$ is greater than demand rate$r_d$;

(c) there is no replenishment (or production) during time $t_2$ and the inventory is decreasing at the rate $r_d$ per unit of time.

Let $Q$ be the number of units produced per order cycle. Then, $t_1 = \frac{Q}{r_p}$

Inventory is building up at the rate of $(r_p - r_d)$

Maximum inventory level $= (r_p - r_d)t$

Minimum inventory level = 0

Average inventory $= \frac{1}{2}\left[(r_p r_d)t_1 + 0\right] = \frac{(r_p - r_d)Q}{2r_p} = \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)$

Ordering cost (or set up cost) $= \frac{D}{Q}C_o$

Carrying cost $= \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$

Total inventory cost is

$$TC = \frac{D}{Q}C_o + \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$$

This cost will be minimum if

$$\frac{D}{Q}C_o = \frac{Q}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$$

$$\frac{d}{dQ}(TC) = \frac{-D}{Q^2}C_0 + \frac{1}{2}\left(1 - \frac{r_d}{r_p}\right)$$

$$\frac{D}{Q^2}C_o = \frac{1}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$$

i.e. $$Q^2 = \frac{2DC_o}{C_h}\left(\frac{r_p}{r_p - r_d}\right)$$

i.e. $Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{r_p}{r_p - r_d}\right)}$

**8.3.2.1. Characteristics of the model**

1. Optimum length of each lot size production run

$$t_1^* = \frac{Q^*}{r_p} = \sqrt{\frac{2DC_o}{C_h r_p \left(r_p - r_d\right)}}$$

2. Optimum number of production runs per year

$$N^* = \frac{D}{Q^*} = \sqrt{\frac{DC_h\left(r_p - r_d\right)}{2C_o r_p}}$$

3. Optimum production cycle time, $t^* = \frac{Q^*}{D}$

4. Total minimum inventory cost

$$TC^* = \frac{D}{Q^*}C_o + \frac{Q^*}{2}\left(1 - \frac{r_d}{r_p}\right)C_h$$

$$= \frac{DC_0\sqrt{C_h\left(r_p - r_d\right)}}{\sqrt{2DC_0 r_p}} + \frac{1}{2}\sqrt{\frac{2DC_o}{C_h}\cdot\frac{r_p}{r_p - r_d}} - \left(1 - \frac{r_d}{r_p}\right)C_h$$

$$= \sqrt{2DC_o C_h\left(1 - \frac{r_d}{r_p}\right)}$$

**8.3.2.2. Example**. A tyre producer makes 1200 tyres per day and sells them at approximately half that rate. Accounting figures show that the production set up cost is Rs. 1000 and carrying cost per unit is Rs. 5. If annual demand is 120000 tyres, what is the optimal lot size and how many production runs should be scheduled per year?

**Solution.** Annual demand, $D$ = 12000 tyres

$C_h$= Rs. 5

$C_o$= Rs. 1000

Production rate, $r_p$= 1200 tyres per day

Demand rate, $r_d$= 600 tyres per day

Optimal lot size, $Q^* = \sqrt{\frac{2DC_o}{C_h} \cdot \frac{rp}{r_p - r_d}}$

$$= \sqrt{\frac{2 \times 120000 \times 1000}{5} \times \frac{1200}{1200 - 600}}$$

$= 2000\sqrt{24} = 9797.96$ = 9798 tyres.

Optimal number of production runs per year

$$N^* = \frac{D}{Q^*} = \frac{120000}{9798} \simeq 13 \text{ runs/year}$$

**8.3.2.3. Example**. A contractor has to supply 10000 paper cones per day to a textile unit. He finds that when he starts production run, he can produce 25000 paper cones per day. The cost of holding a paper cone in stock for one year is 2 paisa and the set up cost of production run is Rs. 18. How frequently should production run be made?

**Solution.** Here $r_d$ = 10000 paper cones per day

$r_p$= 25000 paper cones per day

$D$ = 10000 × 365 = 3,65,0000 cones

$C_H$= Rs. 0.02 per paper cone per year

$C_o$= Rs. 18

Now, $$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{r_p}{r_p - r_d}\right)} = \sqrt{\frac{2 \times 3650000}{0.02} \times \left(\frac{25000}{25000 - 10000}\right) \times 18}$$

$$= \sqrt{3650000 \times 100 \times \frac{25}{15} \times 18} = 104642 \text{ paper cones}$$

Frequency of production runs is given by

$$t^* = \frac{Q^*}{r_d} = \frac{104642}{10000} = 10.46 \text{ days}$$

Thus, production run can be made after every 10.46 days

**8.3.2.4. Exercises.**

1. A product is produced at the rate of 50 items per day. The demand occurs at the rate of 30 items per day. Given that set up cost per order is Rs. 1000 and holding cost per unit time is Rs. 0.05. Find the economic let size and the associated total cost per cycle assuming that no shortage is allowed.

**Answer.** $Q^* = 1732$ units and total inventory cost = Rs. 34.64

**2.** The annual demand for a product is 100000 units. The rate of production is 200000 units per year. The set up cost per production run is Rs. 500 and the variable production cost of each item is Rs. 10. The annual holding cost per unit is 20% of its value. Find the optimum production lot size and the length of the production run.

**Answer.** $Q^* = 10000$ units, $t_1^* = \frac{10000}{200000} = 0.05$years

**3**. An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set-up cost is Rs. 100 per set up and holding cost is Re 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not allowed. Also find the time of cycle, length of each production run, minimum total cost per day and maximum inventory level.

**Answer.** $Q^*$ = 1000 items, Cycle time, $t_l{}^*$= 40 days

Length of each production run, $t_1{}^*$= 20 days, Maximum inventory level = 500 items

Minimum daily cost = Rs. 5

4. A company uses 100000 units of a particular item per year. Each item costs Rs. 2. The production engineering department estimates set up cost at Rs. 25 and the accounting department estimates the holding cost as 12.5% of the value of the inventory per day.

Replenishment rate is uniform 500 units per day.

Assuming 250 working days (per replenishment purpose), calculate

(a) Optimal setup quantity

(b) Total cost on the basis of optimal policy

(c) Optimal number of set up

**Answer.** (a) $Q^*$= 10000 units

(b) Total minimum cost = $TC^* + 200000 = 200500$

(c) Optimum number of set ups $= \frac{D}{Q^*} = 10$

### 8.3.3. Model III: Economic Order Quantity with Different Rates of Demands in Different Cycles

Here the stock will vanish at different time periods with a policy of ordering same quantity for replenishment of inventory.

Here replenishment rate is infinite, replenishment is instantaneous and shortage is not allowed.

The total demand $D$ is specified as demand during total time period $T$ and stock level $Q$ is fixed.

Number of production cycles, $n = \frac{D}{Q}$

Let the demand in different time periods be $D_1, D_2, \ldots, D_n$ respectively so that total demand in time $T$ is

$$D = D_1 + D_2 + \ldots + D_n$$

Where, $T = t_1 + t_2 + \cdots + t_n$

Cost of ordering in time $T$ is given by $\frac{D}{Q} C_o$

Let $C_h$ be the holding cost per item per unit time

Then carrying cost for time $T$ is

$$\frac{Qt_1}{r} C_h + \frac{Qt_2}{2} C_h + \ldots + \frac{Qt_n}{2} C_h$$

$$= \frac{Q}{2} C_h \left(t_1 + t_2 + \ldots + t_n\right) = \frac{Q}{2} C_h T$$

Total inventory cost, $TC = \frac{DC_o}{Q} + \frac{Q}{2} C_h T$

Total cost is minimum, when $\frac{DC_o}{Q} = \frac{Q}{2} C_h T$

i.e. $Q = \sqrt{\frac{2DC_o}{C_h T}} = \sqrt{\frac{2C_o}{C_h} \cdot \frac{D}{T}}$

$$\therefore Q^* = \sqrt{\frac{2C_o}{C_h} \cdot \frac{D}{T}}$$

This result is similar to the result of Model I with the only different that uniform demand is replaced by average demand.

Here, $TC^* = \sqrt{\frac{2D}{T} C_0 C_h}$

and total minimum cost $= \sqrt{\frac{2D}{T} C_0 C_h}$ + cost of material

**Remark**: If $T = 1$ year, then results of this model are exactly same as that of model I.

### 8.3.4. Model IV: Economic Order Quantity when Shortages are Allowed

The assumptions of this model are same as that of model I except that shortages are allowed and shortages may occur regularly. Let $C_s$ be the shortage cost per unit of time per unit quantity.

Ordering cost $= \frac{D}{Q} C_o$

Total inventory over the time period, $t = \frac{1}{2} M t_1$

Average inventory at any time $= \frac{1}{2} M t_1 / t$

Inventory holding cost $= \frac{M t_1}{2t} C_h$

Total amount of shortage over time period $t = \frac{1}{2} S t_2$

Average shortage at any time $= \frac{1}{2} \frac{S t_2}{t}$

Shortage cost $= \frac{1}{2} \frac{S t_2}{t^2} C_s$

From (1),

$-2DC_0 - M^2 C_h + C_s \left( 2Q^2 - 2QM - Q^2 + 2QM - M^2 \right) = 0$

i.e., $C_s Q^2 - M^2 \left( C_h + C_s \right) - 2DC_o = 0$

i.e., $C_s Q^2 - \frac{Q^2 C_s^2}{\left( C_h + C_s \right)^2} \left( C_h + C_s \right) = 2DC_o$ [using (3)]

i.e. $C_s Q^2 - \frac{Q^2 C_s^2}{C_h + C_s} = 2DC_o$

i.e. $Q^2 \left( \frac{C_s C_h + C_s^2 - C_s^2}{C_h + C_s} \right) = 2DC_o$

i.e. $Q^2 = \frac{2DC_o}{C_3 C_h} \left( C_3 + C_h \right)$

$\therefore Q^* = \sqrt{\frac{2DC_o}{C_h} \cdot \frac{C_h + C_s}{C_s}}$

and $M^* = \sqrt{\frac{2DC_0}{C_h} \cdot \frac{C_s}{C_h + C_s}}$

Total minimum cost

$= \frac{D}{Q^*} C_0 + \frac{M^{*2}}{2Q^*} C_h + \frac{\left( Q^* - M^* \right)}{2Q^*} C_s$

$= \sqrt{2DC_o C_h \frac{C_s}{C_h + C_s}}$

Total cost, $TC = \frac{D}{Q} C_o + \frac{1}{2} M \frac{t_1}{t} C_h + \frac{1}{2} S \frac{t_2}{t} C_s$

Using the relationship for similar triangles, we have

$\frac{t_1}{t} = \frac{M}{Q}$ and $\frac{t_2}{t} = \frac{S}{Q}$

$t_1 = \frac{M}{Q} t$ and $t_2 = \frac{S}{Q} t$

So $TC = \frac{D}{Q} C_o + \frac{1}{2} \frac{M^2}{Q} C_h + \frac{\left( Q - M \right)^2}{2Q} C_s$ since $S = a - M$

Since $TC$ is function of two variables $Q$ and $M$, so in order to determine the optimal order size and optimal shortage level, we put $\frac{\partial}{\partial Q}(TC)$ and $\frac{\partial}{\partial M}(TC)$, both equal to zero so that

$$-\frac{DC_o}{Q^2}-\frac{M^2C_h}{2Q^2}+\frac{C_s}{2}\left(\frac{Q\cdot 2(Q-M)-(Q-M)^2\cdot 1}{Q^2}\right)=0 \qquad (1)$$

and

$$\frac{M}{Q}C_h+\frac{2C_s}{2Q}(Q-M)(-1)=0 \qquad (2)$$

From (2),

$$MC_h+C_s(-Q)+MC_s=0$$

$$\Rightarrow M=Q\left(\frac{C_s}{C_h+C_s}\right) \quad (3)$$

**8.3.4.1. Example**. A contractor undertakes to supply Diesel engines to a truck manufacturer at a rate of 20 engines per day. The penalty in the contract is Rs. 100 per engine per day late for missing the scheduled delivery date. The cost of holding an engine in stock for one month is Rs. 150. His production process is such that each month (30 days) he starts producing a batch of engines through the shops and all these are available for supply after the end of the month. Determine the maximum inventory level at the beginning of each month.

**Solution.** Here, $C_h=\text{Rs.}\frac{150}{30}$ per engine per day= Rs. 5 per engine per day

$C_s$ = Rs. 100 per engine per day

$D$ = 20 engines per day

$t^*$ = 30 days

∴ Optimum inventory level at the beginning of each month

$$=M^*=\frac{C_s}{C_h+C_s},Q^*=\frac{C_s}{C_h+C_s}\cdot Dt^*=\frac{100}{5+100}\times 20\times 30=571 \text{ engines}$$

**8.3.4.2. Example**. A dealer supplies you the following information with regard to a product dealt in by him.

Annual demand = 10,000 units

Ordering cost = Rs. 10 per order, Price = Rs. 20 per unit

Inventory carrying cost = 20% of the value of inventory per year

The dealer is considering the possibility of allowing some back order (stock ordered) to occur. He has estimated that the annual cost of back ordering will be 25% of the value of inventory.

(i) What is the optimal number of units of product he should buy in one lot?

(ii) What quantity of product should be allowed to be back ordered, if any?

(iii) What would be maximum quantity of inventory at any time of the year?

(iv) Would you recommended to allow back-ordering? If so, what would be the annual cost saving by adopting the policy of back ordering?

**Sol**. Here, $D$ = 10,000 units

$C_o$ = Rs. 10 per order

$C_h$ = 20% of Rs. 20 = Rs. 4 per unit per year

$C_s$ = 25% of Rs. 20 = Rs. Per unit per year

(i) (a) When stock outs are not permitted:

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2\times 10000\times 10}{4}} = 223.6 \text{ units}$$

(b) When back ordering is permitted

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\cdot\frac{C_h+C_s}{C_s}}$$

$$= \sqrt{\frac{2\times 10000\times 10}{4}\left(\frac{4+5}{5}\right)} = \sqrt{90000} = 300 \text{ units}$$

(ii) Optimal quantity of the product to be back-ordered is given by

$$S^* = Q^*\left(\frac{C_h}{C_h+C_s}\right)$$

$$300\left(\frac{4}{4+5}\right) = 133 \text{ units}$$

(iii) Maximum inventory level, $M^* = Q^* - S^* = 300 - 133 = 167$ units

(iv) Minimum total variable inventory cost in case shortages is not allowed

$$= TC(223.6) = \sqrt{2DC_0C_h}$$

$$= \sqrt{2\times 10000\times 10\times 4} = \text{Rs. } 894.43$$

Minimum total variable inventory cost in case shortage is allowed

= $TC$ (300)

$$= \sqrt{2DC_oC_h\cdot\frac{C_s}{C_h+C_s}} = \sqrt{2\times 10,000\times 10\times 4\cdot\left(\frac{5}{4+5}\right)}$$

= Rs. 666.67

Since $TC$ (300) <$TC$ (223.6), the dealer should accept the proposal for back ordering as this will result in a saving of Rs. (894.43 – 666.67) = Rs. 22776 per year.

**8.3.4.3. Exercise**. A contractor undertakes to supply diesel engines to a truck manufacturer at a rate of 25 engines per day. He finds that the cost of holding a completed engine in stock is Rs. 16 per month and there is a clause in the contract penalizing him Rs. 10 per engine per day late for missing the scheduled delivery date production of engines is in batches and each time a new batch is started, there are setup costs of Rs. 10,000. How frequently should batches be started and what should be the initial inventory level at the time, each batch is completed?

**Solution**. $Q^* = \sqrt{\frac{2DC_o}{C_h} \cdot \frac{C_h + C_s}{C_s}} = \sqrt{\frac{2 \times 25 \times 10,000}{\frac{11}{30}} \left( \frac{\frac{16}{30} + 10}{10} \right)}$ =994 units (app.)

$$t^* = \frac{Q^*}{D} = \frac{994}{25} = 39.76 \simeq 40 \text{ days}$$

**8.3.5. Problems of EOQ with Price Breaks**

In the real world, it is not always true that the unit cost of an item is independent of the quantity procured. Often discounts are offered for the purchase of large quantities. These discounts take the form of price breaks.

Let us consider a manufacturer, who is encountered with a problem of determining an optimum production quantity for each production run and an optimal interval between successive runs. The following conditions are assumed to hold

(i) Demand is known and uniform

(ii) Shortages are not allowed

(iii) Production for supply commodities is instantaneous

Let $Q$ be the lot size in each production run, $D$, total number of units produced or supplied over the time period, $C_0$ the cost per production run, $R$ cost of manufacturing (or purchasing) per unit and $I$ inventory carrying charge expressed as a % of the value of the average inventory. Then total cost is given by

$TC$ = Purchase cost + Holding cost + Ordering cost

$$= Dk + \frac{1}{2}QkI + \frac{D}{Q}C_o$$

$$\frac{d(TC)}{dQ} = \frac{1}{2}kI - \frac{DC_o}{Q^*}$$

$$\frac{d^2}{dQ^2}(TC) = \frac{2DC_0}{Q^3}$$

For maxima or minima, $\frac{d}{dQ}(TC) = 0$

i.e., $\frac{1}{2}kI - \frac{DC_0}{Q^2} = 0$

i.e., $Q = \sqrt{\frac{2DC_o}{kI}}$

At $Q = \sqrt{\frac{2DC_o}{kI}}, \frac{d^2(TC)}{dQ^2} > 0$

Hence total cost is minimum when $Q = \sqrt{\frac{2DC_o}{kI}}$

So total minimum cost, $TC(Q^*) = Dk + \frac{1}{2}Q^* kI + \frac{D}{Q^*}C_o$

$= Dk + \frac{1}{2}\sqrt{\frac{2DC_o}{kI}}kI + \frac{DC_o\sqrt{kI}}{\sqrt{2DC_o}}$

$= Dk + \sqrt{2DC_o kI}$

**8.3.5.1. Purchase Inventory Model with One Price Break**

The purchase inventory model with only one price break may be represented as follows:

| Range of quantity | Purchase cost (per unit) |
|---|---|
| $0 \le Q < b$ | $P_1$ |
| $b \le Q$ | $P_2$ |

Where $b$ is the quantity at and beyond which the quantity discount applies and $P_2 < P_1$

The procedure for obtaining EOQ may be summarized in the following steps:

**Step 1**. Calculate optimum order quantity $Q_2^*$ for the lowest price (highest discount) i.e.

$$Q_2^o = \sqrt{\frac{2DC_o}{P_2 I}}$$

and compare it with the quantity $b$

If $Q_2^o < b$, calculate optimum order quantity $Q_1^o$ for price $P_1$ and compare total inventory cost for $Q = Q_1^o$ with $Q = b$ which given by

$$TC(Q_1^o) = DP_1 + \frac{D}{Q_1^o}C_o + \frac{Q_1^o}{2}IP_1$$

$$TC(b) = DP_2 + \frac{D}{b}C_o + \frac{b}{2}IP_2$$

If $TC(Q_1^o) > TC(b)$, then $Q^o = b$ otherwise $Q^o = Q_1^o$.

**8.3.5.1.1. Example**. Find the optimum order quantity for a product for which the price breaks are as follows:

| Quantity | Unit cost (Rs.) |
|---|---|
| $0 \le Q < 500$ | 10.00 |
| $500 \le Q$ | 9.25 |

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and ordering cost is Rs. 350.

**Solution**. We are given

$C_o$ = Rs. 350

$D$ = 200 units per month

$I$ = 2% = 0.02

$P_1$ = Rs. 10

$P_2$ = Rs. 9.25

Highest discount available is Rs. 9.25. So we compute $Q_2^o$ as

$$Q_2^o = \sqrt{\frac{2DC_o}{IP_2}} = \sqrt{\frac{2\times 200\times 350}{0.02\times 9.25}} = 870 \text{ units}$$

Since $Q_2^o > b = 500$, the optimum purchase quantity is given by $Q^o = Q_2^o$ = 870 units.

**8.3.5.1.2. Example**. Find the optimal order quantity for a product having the following characteristics:

Annual demand = 2400 units

Ordering cost = Rs. 100

Cost of storage = 24% of the unit cost

**Price break**

| Quantity | Unit cost (Rs.) |
|---|---|
| $0 \le Q < 500$ | 10 |
| $500 \le Q$ | 9 |

**Solution.** We have $D$ = 2400 units per year

$I$ = 0.24

$C_o$ = Rs. 100

$P_1$ = Rs. 10

$P_2$ = Rs. 9

Highest discount available is Rs. 9, so we compute $Q_2^*$ as

$$Q_s^* = \sqrt{\frac{2DC_o}{P_2 I}} = \sqrt{\frac{2\times 2400\times 100}{9\times 0.24}} = 471 \text{ units}$$

Since $Q_2^* < 500$, $Q_2^*$ is not feasible

We calculate $Q_1^*$ as

$$Q_1^* = \sqrt{\frac{2DC_o}{P_1 I}} = \sqrt{\frac{2 \times 2400 \times 100}{10 \times 0.24}} = 447 \text{ units.}$$

Now total cost corresponding to order size 447 is

$$TC\left(Q_1^*\right) = DP_1 + \frac{D}{Q_1^*} C_o + \frac{Q_1^*}{2} IP_1$$

$$= 2400 \times 10 + \frac{2400 \times 100}{447} + \frac{447}{2} \times 0.24 \times 10$$

= 2400 + 536.91 + 536.4 = Rs. 25073.31

Total cost at price break is

*TC* (*b*) = *TC* (500)

$$= DP_2 + \frac{D}{b} \times C_o + \frac{b}{2} IP_2$$

$$= 2400 \times 9 + \frac{2400}{500} \times 100 + \frac{500}{2} \times 0.24 \times 9$$

= Rs. 22620

Since $TC(b) < TC\left(Q_1^*\right)$, the optimal order quantity is the price discount quantity is 500 units.

#### 8.3.5.2. EOQ Problems with Two Price Breaks

When there are two price breaks i.e. two quantity discounts, the situation can be represented as

| Range of quantity | Purchase cost per unit |
|---|---|
| $0 \le Q < b_1$ | $P_1$ |
| $b_1 \le Q < b_2$ | $P_2$ |
| $b_2 \le Q$ | $P_3$ |

where $b_1$ and $b_2$ are the quantities which determine this price discount. The procedure for obtaining EOQ may be summarized in the following steps:

1. Compute the optimal order quantity for the lowest price. Let it be $Q_3^*$.
2. If $Q_3^* \ge b_2$, the optimum order quantity is $Q_3^*$.
3. If $Q_3^* < b_2$, calculate $Q_2^*$, the optimal order quantity for the next lowest price.
4. If $b_1 \le Q_2^* < b_2$, then compare $TC\left(Q_2^*\right)$ and $TC\left(b_2\right)$ to determine the optimum purchase quantity.
5. If $Q_2^* < b_1$, calculate $Q_1^*$ and compare $TC\left(b_1\right)$, $TC\left(b_2\right)$ and $TC\left(Q^*\right)$ to determine the optimum, purchase quantity.

**8.3.5.2.1. Example**. Find the optimal order quantity for a product for which the price discounts are as:

| Range of quantity | Unit price (Rs.) |
|---|---|
| $0 \le Q < 500$ | 10.00 |
| $500 \le Q < 750$ | 9.25 |
| $750 \le Q$ | 8.75 |

The monthly demand for the product is 200 units, storage cost is 2% of unit cost and ordering cost is Rs. 100.

**Solution.** Here $D$ = 200 units

$I$ = 2% of unit cost

$C_0$ = Rs. 100 per order

$P_1$ = Rs. 10

$P_2$ = Rs. 9.25

$P_3$ = Rs. 8.75

We calculate $Q_3^*$ as

$$Q_3^* = \sqrt{\frac{2DC_o}{IP_3}} = \sqrt{\frac{2\times200\times100}{8.75\times0.02}} = 475 \text{ units}$$

Since $Q_3^* < b_2 = 750, Q_3^*$ is not feasible.

Now $Q_2^* = \sqrt{\frac{2DC_o}{IP_2}} = \sqrt{\frac{2\times200\times100}{0.02\times9.25}} = 465$ units

Since $Q_2^* = b_1 = 500$, we compute $Q_1^*$

$$Q_1^* = \sqrt{\frac{2DC_o}{IP_1}} = \sqrt{\frac{2\times200\times100}{0.02\times10}} = 447 \text{ units.}$$

Now,

$$TC\left(Q_1^*\right) = DP_1 + \frac{D}{Q_1^*}C_o + \frac{Q_1^* IP_1}{2}$$

$$= 200\times10 + \frac{200}{447}\times100 + \frac{447}{2}\times0.02\times10$$

= 2000 + 44.74 + 44.70 = Rs. 2089.44

$$TC\left(b_1\right) = TC\left(500\right)$$

$$= DP_2 + \frac{D}{b_1}C_o + \frac{b_1}{2}IP_2$$

$$= 200\times9.25 + \frac{200}{500}\times100 + \frac{500}{2}\times0.02\times9.25$$

$= 1850 + 40 + 46.25$ = Rs. 1936.25

$$TC\left(b_2\right) = TC\left(750\right)$$

$$= DP_3 + \frac{D}{b_2}C_o + \frac{b_2}{2}IP_3$$

$$= 200 \times 8.75 + \frac{200}{750} \times 100 + \frac{750}{2} \times 0.02 \times 8.15$$

= 1750 + 26.67 + 65.62 = Rs. 1842.29

The lowest total inventory cost is Rs. 1842.24

So optimal order quantity is $Q^* = b_2 = 750$

### 8.3.5.3. Purchase Inventory Model with *n* Price Breaks

When there are *n* price breaks, the situation may be illustrated as

| Range of quantity | Purchase cost per unit |
|---|---|
| $0 \le Q < b_1$ | $P_1$ |
| $b_1 \le Q < b_2$ | $P_2$ |
| $\vdots$ | $\vdots$ |
| $b_{n-1} \le Q$ | $P_n$ |

where $b_1, b_2, \ldots b_{n-1}$ are the quantities which determine the price breaks. Let $Q_1^*, Q_2^*, \ldots, Q_n^*$ be EOQ corresponding to prices $P_1, P_2, \ldots, P_n$ respectively. The procedure for obtaining optimum order quantity is:

1. Compute $Q_n^*$. If $Q_n^* \ge b_{n-1}$, then the optimum purchase quantity is $Q_n^*$.
2. If $Q_n^* < b_{n-1}$, then compute $Q_{n-1}^*$.
   If $Q_{n-1}^* \ge b_{n-2}$, then optimum order quantity is determined by comparing $TC\left(Q_{n-1}^*\right)$ with $TC\left(b_{n-1}\right)$.
3. If $Q_{n-1}^* < b_{n-2}$, compute $Q_{n-2}^*$. If $Q_{n-2}^* \ge b_{n-3}$, then optimum order quantity is determined by comparing $TC\left(Q_{n-2}^*\right)$ with $TC\left(b_{n-2}\right)$ and $TC\left(b_{n-1}\right)$.
4. If $Q_{n-2}^* < b_{n-3}$, compute $Q_{n-3}^*$
   If $Q_{n-3}^* \ge b_{n-4}$, then optimum order quantity is determined by comparing $TC\left(Q_{n-3}^*\right)$ with $TC\left(b_{n-3}\right)$, $TC\left(b_{n-2}\right)$ and $TC\left(b_{n-1}\right)$.
5. Continue in this way until $Q_{n-j}^* \ge b_{n-(j+1)}; 0 \le j \le n-1;$ and then compare $TC\left(Q_{n-j}^*\right)$ with $TC\left(b_{n-j}\right)$ , $TC\left(b_{n-j+1}\right), TC\left(b_{n-j+2}\right), \ldots, TC\left(b_{n-1}\right)$.

This procedure involves a finite number of steps.

**8.3.5.3.1. Example**. The annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost. The ordering cost is Rs. 180 for each order. The unit cost depends upon the amount ordered. The range of amount ordered and the unit cost price are as follows:

| Range of amount ordered | Price per unit |
|---|---|
| $0 \le Q < 500$ | Rs. 25 |
| $500 \le Q < 1500$ | Rs. 24.80 |
| $1500 \le Q < 3000$ | Rs. 24.60 |
| $3000 \le Q$ | Rs. 24.40 |

Find the optimal order quantity.

**Solution.** Here $D$ = 500 units

$C_o$ = Rs. 180 per order

$I = 0.10$

$b_1 = 500,\ b_2 = 1500,\ b_3 = 3000$

$P_1$ = Rs. 25, $P_2$ = Rs. 24.80

$P_3$ = Rs. 24.60, $P_4$ = Rs. 24.40

**Step 1** $Q_4^* = \sqrt{\dfrac{2DC_o}{IP_4}} = \sqrt{\dfrac{2\times 500\times 180}{0.10\times 24.40}} = \sqrt{\dfrac{1000\times 10\times 18000}{2440}}$

$= 1000\sqrt{\dfrac{180}{2440}} = 271.6 \simeq 272$ units

Since $Q_4^* < b_3$, we compute $Q_3^*$.

**Step 2** $Q_3^* = \sqrt{\dfrac{2DC_o}{IP_3}} = \sqrt{\dfrac{2\times 500\times 180}{0.16\times 24.60}}$

$= 100\sqrt{\dfrac{18}{246}} = 270$ units

Since $Q_3^* < b_2$, we compute $Q_2^*$

**Step 3** $Q_2^* = \sqrt{\dfrac{2DC_o}{IP_2}} = \sqrt{\dfrac{2\times 500\times 180}{0.10\times 24.80}}$ = 269 units

Since $Q_2^* < b_1$, we compute $Q_1^*$

**Step 4** $Q_1^* = \sqrt{\dfrac{2\times 500\times 180}{0.10\times 25}} = \sqrt{\dfrac{180}{25}\times 10000} = \sqrt{72000}$ = 268 units.

Now we compute $TC\left(Q_1^*\right), TC\left(b_1\right), TC\left(b_2\right), TC\left(b_3\right)$ and compare them to get optimal order quantity.

$$TC\left(Q_1^*\right) = DP_1 + \frac{D}{Q_1^*}C_o + \frac{1}{2}Q_1^* IP_1$$

$$= 500\times 25 + \frac{500}{268}\times 180 + \frac{268}{2}\times 0.10\times 25$$

= 12500 + 335.82 + 335 = Rs. 13170.82

$$TC(b_1) = DP_2 + \frac{D}{b_1}C_o + \frac{1}{2}b_1 IP_2$$

$$= 500 \times 24.80 + \frac{500}{500} \times 180 + \frac{500}{2} \times 0.10 \times 24.80$$

= 12400 + 1800 + 620 = Rs. 14820

$$TC(b_2) = DP_3 + \frac{D}{b_2}C_o + \frac{1}{2}b_2 IP_3$$

$$= 500 \times 24.60 + \frac{500}{1500} \times 180 + \frac{1}{2} \times 1500 \times 0.10 \times 24.60$$

= 12300 + 60 + 1845 = Rs. 14205

$$TC(b_3) = DP_4 + \frac{D}{b_3}C_o + \frac{1}{2}b_3 IP_4$$

$$= 500 \times 24.40 + \frac{500}{3000} \times 180 + \frac{1}{2} \times 3000 \times 0.10 \times 2440$$

= 12200 + 30 + 3660 = Rs. 15890

Since $TC(Q_1^*) < TC(b_2) < TC(b_1) < TC(b_3)$,

Optimum order quantity is $Q_1^*$ i.e. 268 units.

## 8.4. CHECK YOUR PROGRESS

1. Why inventory is maintained?
2. Explain the meaning of inventory control.
3. Explain the inventory model when demand rate is uniform.
4. Derive EOQ model for an inventory problem when shortages are allowed.

## 8.5. SUMMARY

Inventory may be defined as stock of idle resources that are stored or reserved in order to ensure smooth and efficient running of business affairs as inventories are essential for almost all business. In this chapter, we explained the concept of inventory control and EOQ. The models discussed in this chapter are:

(i) EOQ Model with Uniform Demand,

(ii) EOQ Model with uniform replenishment,

(iii) EOQ with different rates of demands in different cycles,

(iv) EOQ Model when shortages are allowed, and

(v) EOQ Model with price breaks