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Unit-II

Demography

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UNIT-II

Mortality and its Measures

Mortality is defined as the demographic event of death. Since death is a biological phenomenon that occurs just once to each individual, the analysis is simpler than, say, the study of fertility wherein the event of birth can occur with varying frequency among women. Mortality analysis begins with good quality data on deaths and population. These data are conventionally obtained from vital registration systems and population censuses respectively. The crude death rate and the specific death rates (age, sex, age-sex, age sex-cause of death specific) are simple measures of mortality. The other measures are based on the life tables.

Measures of Mortality

Crude Death Rate The crude death rate is calculated by dividing the number of registered deaths in a year by the mid-year population for the same year. The rate is expressed as per 1,000 population.

$$\text{Crude Death rate} = \frac{\text{total number of deaths}}{\text{total mid} - \text{year population}} \mathbf{x\ 1000}$$

This rate has a simple interpretation, for it gives the number of deaths that occur, on the average, per 1,000 people in the community. Further, it is relatively easy to compute, requiring only the total population size and the total number of deaths. Besides, it is a probability rate in the true sense of the term. It represents an estimate of the chance of dying for a person belonging to the given population, because the whole population may be supposed to be exposed to the risk of dying of something or the other. However, it has also some serious drawbacks. In using the CDR, we ignore the fact that the chance of dying is not the same for the young and the old or for males and females, and the fact that it may also vary with respect to race, occupation or locality of dwelling.

Specific Death Rate

The crude death rates for specific causes of death are calculated in a similar way by selecting deaths due to specific cause as the numerator and mid-year population as the denominator. Thus,

$$\text{Cause- specific death rate} = \frac{total\ number\ of\ deaths\ to\ some\ particular\ cause}{total\ mid-year\ population}$$

The rates could be made specific to sex by selecting the numerator and the denominator for each sex of the population.

Age Specific Death Rates (ASDR)

The age-specific death rates are calculated from deaths and population both specific to each age (or age group) of the population. Thus,

$$\text{Age Specific Death Rate} = {}_nD_x / {}_nP_x \times 1000$$

where 'x' indicates the age and 'n' the class interval of age. The age-cause-specific death rates are obtained by selecting deaths in specific age and cause group of the population as the numerator It should be noted that the sum of the cause-specific rates over all causes equals the crude death rate. Similarly, the sum of the age-cause-specific death rates equals the age-specific death rate at a given age.

Standardization is a technique, which provides a summary measure of the rates (similar to the crude rates) while controlling for the compositional variation between the populations being compared. Thus, a comparison of the standardized rates gives a 'true' comparison of the phenomenon studied. We shall illustrate the calculations of the standardized rates with the help of the death rates.

The ASDR is a type of central death rate, that is, a rate relating to the events in a given category during a year to the average population of the category. In a high mortality situation, the death rates by age, that is, the age specific death rates, form a U-shaped curve indicating a high mortality in early and old ages. At low levels of mortality, the pattern of ASDR changes to J-shaped indicating a relatively higher mortality in the very early period of life, which drops to a low level after the hazards of early life and extends over a long periodof life, and finally it rises sharply in old ages.

Standardized Death Rate

Method of Direct Standardization

A. In this method the distributions of the compositional variables (age, sex, marital status etc.) of the populations that are being compared, are made identical and the standardized rates (similar to the crude rates) are calculated such that the difference between them is only due to the variation in the age-specific rates of their population.

A Standard population is selected which is employed for deriving all the standardized rates in a set to be compared.

Data Needed

(1) For one compositional variable (say age) standardization, age distribution of the standard population, and

(2) Age-specific death rates in all populations to be compared.

Calculations

If M(i, x) represents the age-specific death rate at age (i) for population (x), and P (i,s) is the standard population at age (i), P(s) is the total standard population.

The standardized death rate for population 'x'= $\boldsymbol{\Sigma} \frac{(i,)}{P(s)}\mathbf{.M(i,x)}$

$$\textbf{or} = \frac{\Sigma(i,s).M(i,x)}{P(s)}$$

The numerator is the number of expected deaths in the standard population had the age-specific death rates of population (x) applied to the standard population, and the denominator is the total standard population. The rate is multiplied by 1,000 to express the rate as per 1,000 population. (All the calculations are done with the rates per person. Finally, the standardized death rate is multiplied by the constant 1,000).

B. If the standardized death rate is required after controlling for the two characteristics of the population, say age and sex, the data needed will be the same as on the previous page but split by sex as well. Thus, the standardized death rate for population x will be:

$$\frac{\Sigma[P(i,s,males).M(i,males)+P(i,s,females.M(i,females)]}{[P(s,males)+P(s,females)]}$$

C. If the death rates of males and females are to be compared, these are two different populations, and the method given under A is to be used. Thus,

$$\text{The standardized death rate of males} = \frac{\Sigma[P(i,s).M(i,males)}{P(s)}$$

$$\text{The standardized death rate of females} = \frac{\Sigma[P(i,s).M(i,females)}{P(s)}$$

Method of Indirect Standardization

This method is used when the age-specific death rates for the populations to be compared cannot be calculated because of the distribution of the number of deaths by age is unavailable or not reliably available, but the total number of deaths and the age distribution of populations whose rates are to be compared are available.

Data Needed

(1) Observed number of deaths in all populations whose death rates are to be compared.

(2) Age distribution of all populations whose death rates are to be compared.

(3) Age-specific death rates for a population to be used as standard.

(4) Crude death rate in the standard population.

Calculations

A. If P(i,x) represents the population 'x' at age (i), M(i,s) is age-specific death rate at age 'i' in standard population, M(s) is the crude death rate in the standard population, O(x) is the observed number of deaths in population 'x', and E(x) is the expected deaths in population 'x', then

$$\text{The standardized death rate for population x} = \{o(x) / E(x)\}. M(s)$$

$$\text{The expected deaths in population x} = E(x) = \Sigma P(i,x).M(i,s)$$

(All the calculations are done with the rates per person. Finally, the standardized death rate is multiplied by the constant 1,000).

Life Table:

A life table in demography is a statistical tool used to summarize and analyze the mortality and survival patterns of a population over time. It provides a detailed breakdown of the probability of death and other demographic indicators at different ages, usually in one-year age intervals. Life tables are widely used in the fields of demography, epidemiology, and actuarial science. The key components of a life table include:

- **Age (x)**: This represents the age intervals, typically in single years, starting from birth (age 0) and continuing until the last age of interest.
- **l_x (Survivorship)**: This is the number of individuals surviving to the beginning of age x out of the initial cohort born at the start of the observation period. It represents the number of survivors at the beginning of each age interval.
- **q_x (Mortality Rate)**: This is the probability of dying between age x and x + 1, given survival to age x. It is calculated as the number of deaths during the interval divided by the number of survivors at the beginning of the interval (qx = dx / lx).
- **p_x (Survival Probability)**: This is the probability of surviving to at least age x. It is calculatedas the product of the survival probabilities up to that age (px = l(x + 1) / lx).
- **T_x (Total Years Lived):** This is the total person-years lived by the cohort up to age x, which is calculated as the sum of lx.
- **e^x (Life Expectancy at Age x)**: This represents the average number of additional years a person of age x can expect to live, calculated as the remaining years of life divided by the number of survivors at age x (e^x = Tx / lx).

CONSTRUCTION OF LIFE TABLE:

Construction of a life table means computation of certain age-wise life table functions, if ASDRs are available or provided. A complete life table consists of eight columns for eight life table functions as
mentioned below:

x: The exact age in completed years.

l_x: The number of survivors at the exact age x out of the initial cohort. It is generally taken to be 1,00,000. Further, the number of survivors at age 1, l1, obviously be equal to l1 = l0 – d0; where d0 is the number of deaths occurred at age 0. Therefore, we see that in general, lx will be obtained by

$$l_{x+1} = l_x + d_x \qquad \text{for } x=0,1,2.....$$

d_x: The number of deaths in the age interval x to (x + 1) in the initial cohort. It gives the number of those who could celebrate their xth birthday but not the (x + 1)th birthday. From the above, we have a relation between lx and dx functions which is

$$d_x = l_x + l_{x+1} \qquad \text{for } x=0,1,2.....$$

q_x: Probability that a person living at the age x will die before reaching age (x+1). It is the proportion of persons dying between the ages of x and (x +1) to the number of persons alive at the age of x, that is, at the beginning of the interval. It is given by the formula

$$q_x = \frac{d_x}{l_x} \Rightarrow d_x = l_x q_x \text{ for all x.}$$

In life tables, q_x function plays a very important role, since it is the key function, using which we are able to compute all other functions of life tables.

p_x: Probability of surviving between age x and (x + 1). It is the same as the proportion of persons surviving up to the end to the interval, that is, at age (x + 1) years to the number of persons alive at the beginning of the interval, that is, at age x.

Obviously, we observe that $p_x = 1 - q_x$

L_x: It may be interpreted as the total number of years lived by the cohort belonging to age group x to (x + 1). It is given by

$$L_x = \frac{l_x + l_{x+1}}{2} = l_x - \frac{1}{2} d_x$$

T_x: It is the total number of years lived by the cohort while at age x and thereafter. It is given by

$$T_x = L_x + L_{x+1} + \ldots = L_x + T_{x+1}$$

e^0_x : The expectation of life at age x. In fact, it is the average remaining lifetime. It gives the average number of years a person of age x is likely to survive under the existing mortality rate. It is given by

$$e^0_x = \frac{T_x}{l_x}.$$

USES OF LIFE TABLE'S INTERPRETATION:

Life table is a hypothetical model which can be used to understand and predict the mortality behavior over ages in a human population and some other important characteristics related to 'deaths'; one of the most crucial vital events in the life of an individual.

Some of the important uses of life tables are as follows:

- Life tables are of maximum utility to actuaries to work out the rate of premium in life insurance policies of different kinds for persons of different age groups, as it provides the life expectancies of persons at different ages experiencing a particular type of mortality behavior.
- If demand appears, age-wise as well as sex-wise population projections for some time in future may be done on the basis of available life table parameters. The life table for any specific section of the society can be prepared and used to deduce many conclusions about population growth, specific death rate, etc., of that section of the society.
- A life table clearly depicts the distribution of survivals and deaths of people at and after certain age and many other facts related to mortality behavior of the community. No doubt, similar to life tables, hypothetical models may also be prepared for some other aspects of human populations over age and time, like, educational status, employment status, availability of number of person for labour sectors, military sectors, etc., on the basis of some observed factsrelated to these Characteristics.
- If life table values for a specific community and region is available, it often helps social scientists to assess the accuracy of census figures, death and birth registrations which are thought to be affected by many kinds of census errors, such as, investigator's bias, respondent's bias, age-reporting bias, lack of memory bias, etc.

- The computation of measure of intrinsic natural increases, such as net production rate and the true rate of natural increase, makes use of life tables.
- It helps to evaluate the impact of family planning on population growth.

Governments of different countries or different segments of a country often use life table functions of the corresponding life table for making policies and programmes relating to health behaviour of their people. This helps them to access how far the new scientific inventions, sophisticated medical treatments and better living conditions have increased the span of life.

- Estimates of migration can also be made from life tables.

FORCE OF MORTALITY:

The force of mortality is a concept used in demography to describe the instantaneous rate at which individuals are dying at a particular age. It is denoted by the symbol μ_X, where x represents age. The force of mortality is also sometimes referred to as the hazard rate or the death rate.

Mathematically, the force of mortality is defined as the limit of the probability of dying in a very small age interval divided by the width of that interval as the interval approaches zero:

$$\mu_X = \lim_{\delta x \to 0} \frac{Pr[x \leq T < x+\delta x | T \geq x]}{\delta x}$$

Here, T represents the random variable denoting the time of death, and $[x \leq T < x + \delta x | T \geq x]$ is the conditional probability of dying in the small interval [x,x+δ x) given survival to age x.

The force of mortality is often used in the context of survival analysis and life insurance. It provides a more dynamic measure of mortality risk than traditional life tables, asit focuses on the instantaneous risk of death at a specific age. Understanding the force of mortality is crucial for making mortality predictions, setting life insurance premiums, and making informed decisions in fields related to population studies and risk management.

ABRIDGED LIFE TABLE:

If Ages Are Not Recorded In Single Years Of Age But They Are Grouped Into Class Intervals (X To X +N) For N ($\neq$ 1) = 2, 3, ..., We Cannot Construct A Complete Life Table For Obvious Reasons. The Life Table Constructed With The Grouped Age

Intervals In The Form (X, X + N); X = 0, 1, 2, ...; N ($\neq$ 1) = 2, 3, ..., Is Known As "Abridged Life Table". In Fact, An Abridged Life Table Is A Compulsion To Prepare When The Asdrs In The concerned population are not available for each age; rather they are available for age groups with some specific intervals, like, for 5-year age intervals, 10-year age intervals, etc.

However, sometimes even if the ASDRs are available for each year, to avoid unnecessary complications in computation as well as to minimize the time spent, abridged life tables are prepared instead of complete life table so as to get life table functions only at some specific ages. For example, one may be interested to know the values of life table functions only at ages 0, 5, 10, ... years and not for ages between them. For this purpose, he may pick-up ASDRs at ages 0, 5, 10 years from the given year-wise mortality rates for the community. Another method of constructing an abridged life table is through the condensation of a complete life table rather than through the omission of some of its rows as described in the first method.

Since, the abridged life tables are constructed for age-groups of some specific intervals; we cannot use same symbols for life table functions as we used in case of complete life tables. Instead of these, we use symbols x, ${}_nl_x$, ${}_nd_x$, ${}_nq_x$, ${}_np_x$, ${}_nL_x$, ${}_nT_x$ and ${}_n e^0_x$ for functions if the age groups are denoted by

(x, x +n) for n = 2, 3, 4, ...For example, ${}_3d_x$ and ${}_5q_x$, respectively show the value of the function, d, for the age group (x, x +3) and value of the function, q, for the age interval (x, x +5).

GROSS REPRODUCTION RATE (GRR)

Fertility rates include the birth of children of both the sexes. But the population growth depends mainly on the birth of female children who are the future mothers. Hence, population growth is mainly a function of the fertility rate, restricted to female children. The demographic year book published by United Nations, 1954, has defined GRR as, "The gross reproduction rate indicates the average number of daughters who would be born to a group of girls beginning life together in a population where none died before the upper limit of child bearings age and where the given set of fertility rates was in operation."

If ${}^{f}S_X$ denotes the fertility rate at age x, restricted only to the births of the female infants, the function is $\sum_{x=w1}^{w2}$ called GRR, where w1 and w2 are respectively the lower GRR is based on the following assumptions:

(i) There is no mortality of newly born female children till they attain the highest reproductive age. In Indian situation, this is taken to be 49 years.

(ii) There are no gains or losses in the population due to migration.

(iii) The current fertility rate is maintained till their highest child-bearing age.

Due to these assumptions which are very rare for a human population, GRR is considered to be a hypothetical figure. However, even then, it is measured as a measure of population growth.

In other words, the GRR is the sum of age specific birth rates of women of child-bearing age restricted to female births only. However, computation of GRR with this formula requires births classified according to age of mother and according to sex. Then another working formula for GRR is derived as follows:

An approximate value of GRR can be obtained by assuming that the sex ratio at birth, that is, the ratio of the number of male births to the number of female births remained constant over all the ages of mother. This means that here we shall have approximately

$$\frac{fBx}{B_x} = \text{a constant, say, K;}$$

where ${}^{f}B_x$ and B_x respectively denote the female birth at age x and total birth at age x. Then, we have

$$k = \frac{\sum_{w_1}^{w_2}}{\sum_{w_1}^{w_2}} \frac{fBx}{B_x} = \frac{fB}{B}$$

so that ${}^{f}B_x = B_x \frac{fB}{B}$ and ${}^{f}i_x = i_x \cdot \frac{fB}{B}$

Therefore, an estimate of the GRR will be given by

$$\text{GRR} = \frac{fB}{B} \sum_{w_1}^{w_2} i_x \text{ is the Total Fertility Rate (TFR).}$$

This indicates that the GRR may also be calculated using the above working formula, which is

$$GRR= TFRx\frac{No.of\ FemaleBirths}{totalbirts}$$

Net Reproduction Rate (NRR)

It is clear from the description of gross reproduction rate that it has its significance in relation to the replacement of one generation by the next one. This replacement may be towards increasing the population or decreasing it. But the GRR overestimates the next generation as the loss due to mortality was ignored and it was assumed that all the new born female children attain their maximum reproductive age. This made the GRR quite artificial. Of course, other losses like migration, unmarried persons, etc., do affect the next generation, but these factors are not so important as mortality. Hence, by making use of the life table, it is possible to make some allowance for mortality losses and obtain the reproduction rate which is devoid of the losses due to mortality. The net reproduction rate, as defined in the Demographic Year Book, United Nations, 1954 is interpreted as the average number ofdaughters that would be produced by women throughout their life-time if they were exposed at each age to the fertility and mortality rates on which the calculation is based."

From the above discussion, it is evident that NRR measures the extent to which the female infants, who continue to survive their maximum reproduction age, can reproduce infants of the same sex. The formula for net reproduction rate is,

$$NRR = \sum_{all\ age\ groups} (\frac{B_r}{B})\text{x class interval of the age group}$$

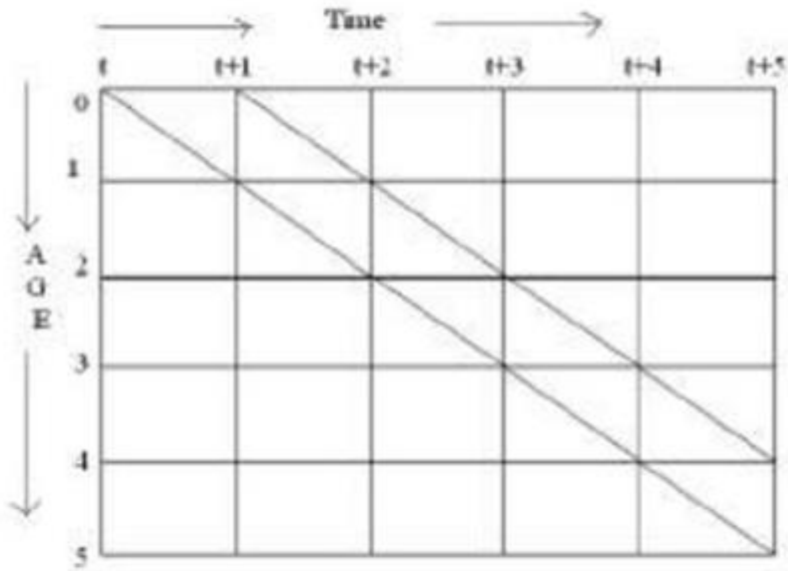
where,

Bg :No. of female infants born to women of specific reproduction age groups.

B: No. of women in each specific reproductive age group.

S: Survival rate p_x per woman, which is obtained from the life table.

Cohort fertility analysis lexis diagram



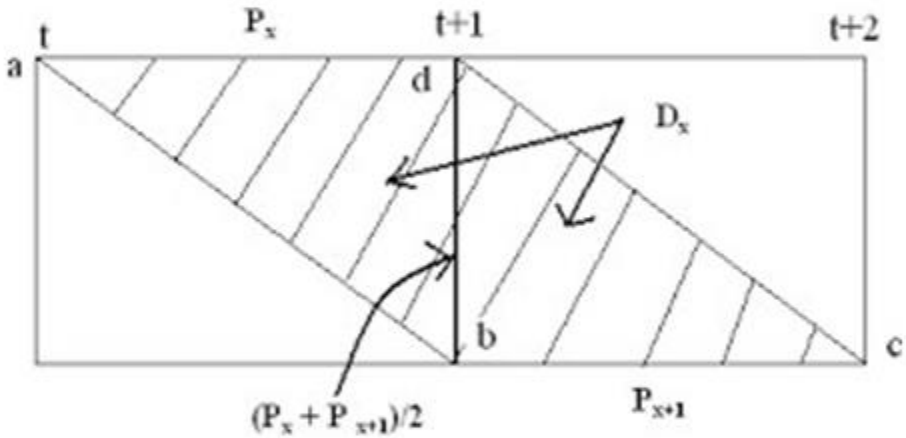
THE LEXIS DIAGRAM

Lexis diagram is a two dimensional diagram used to characterize events that occur to individuals belonging to different cohorts. In this diagram time is considered along the horizontal axis and age along vertical axis. Each individual member of a population is represented by a line at 45^0 of either axis starting at age x = 0 and at the moment of birth and terminating at a point which corresponds to both his age and time of death. A part of diagram can be represented as follows:

In the Lexis diagram, the time of occurrence of event is shown on the X- axis and the duration since an initial reference data (eg. Age since birth) on the Y-axis. In the diagram above, time refers to the exact time of occurrence, and age, the exact age of the person.

CALCULATION OF THE COHORT RATES

The rates for the cohorts are usually based on the probability concept. However, cross-section rates for the cohorts (Known as Central rates) are also calculated. Consider the following Lexis diagram.



If we know the population at exact age X during time t- t+1 (say PX) and at exact ageX+1 (say PX+1) during time t+1- t+2, and the deaths occurring to this cohort in the parallelogram abcd (say DX),

The probability of dying between exact ages X and X+1 will be$=\frac{D_x}{P}$

The central death rate at age X will be $= \frac{D_x}{p_x+P_{x+1}/2}$

(p_x+P_{x+1}) 2 will be the average or mid - year population (represented by the vertical line 'db') on the assumption that the deaths have occurred uniformly over the parallelogram 'abcd'. However, this is not true for deaths, which occur at very early ages, especially among the infants in their first 12 months of life. The mid-year population is estimated by the use of the separation factors.

Fertility models:

Models of fertility find application in many of the same areas of demographic estimation as models of mortality. They can be used to smooth a set of observed fertility rates or to convert a schedule classified in five-year age groups into a schedule by single years of age. They also find widespread use in population projections and various approaches to demographic estimation from limited and defective data.

In this session, we describe two different approaches to modeling fertility:

1. The first makes use of parametric models (the analogue of, for example, the Makeham or Heligman-Pollard mathematical models of mortality rates).
2. The second approach makes use of a fertility standard that can be used in a relational

model of fertility, much as standard life tables can be used in relational models of mortality.

Fertility levels and patterns

To be useful, a fertility model, whether parameterized or relational, should havecertain properties. First, and most obviously, the model should – ideally – be capable of representing the wide variety of fertility patterns that are observed in practice. In the next few pages, we explore what is meant by different levels and patterns of fertility. We then consider what properties a good model of fertility should exhibit.

By the level of fertility, we conventionally mean the total fertility rate (TFR) in a population.

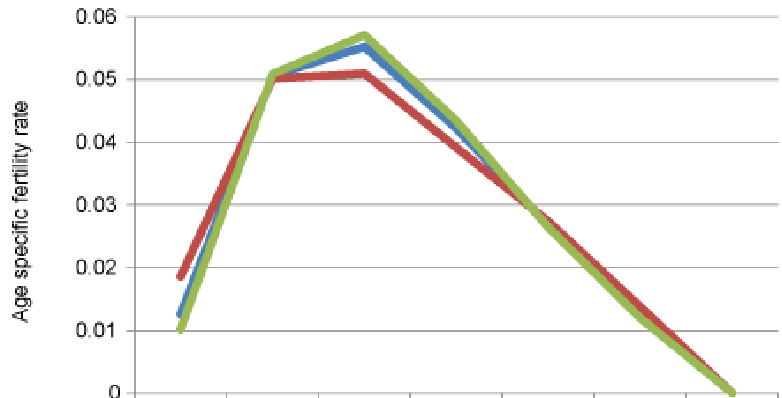
Parametric models of fertility

In this section, we discuss four parametric distributions and models that have been used to describe fertility patterns. These are

- The UN model fertility schedule
- The Brass polynomial
- Romaniuk's three-parameter fertility model
- The Hadwiger fertility model

The UN model schedule

One of the earliest, and certainly the simplest, fertility model is that produced in 1956 by the United Nations (United Nations 1956). It is an empirical model, based on the average shape of fertility distributions in 52 countries, 15 with comparatively high fertility (although the country with the highest fertility had a TFR of just over 5 children per woman), and 37 with relatively low fertility. With the possible exception of data from two Spanish protectorates in North Africa (Ceuta and Melilla), the only data from Africa were from colonial immigrant populations, while data from only three Latin American countries were included. This schedule is therefore barely representative of developing country fertility patterns. The fertility schedules are presented in the figure below.



	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Combined schedule	0.0126	0.0506	0.0552	0.0422	0.0268	0.0126	0.0000
High fertility	0.0186	0.0501	0.0509	0.0392	0.0274	0.0138	0.0000
Low fertility	0.0102	0.0509	0.0571	0.0434	0.0264	0.0120	0.0000

Brass' fertility polynomial

Around 1962, Brass devised a parametric model - his fertility polynomial. He used this widely in developing many of his indirect estimation methods. The polynomial is described in greater detail in Brass (1975).

In this model, the age-specific fertility rate, f(x) at exact age x is given by:

$$f(x)=c(x-s)\ (s+33-x)^2 \text{ for } \quad s<x<s+33$$

Since the integral of the f(x) values between s and s+33 is the TFR, we can express c in terms of the TFR:

$$TFR=\int_{s}^{s+33} f(x)\,dx=\int_{s}^{s+33} c(x-s)\ (s+33-x)^2\,dx$$

$$c=\frac{TFR}{\int_{s}^{s+33} c(x-s)\ (s+33-x)2\ dx}$$

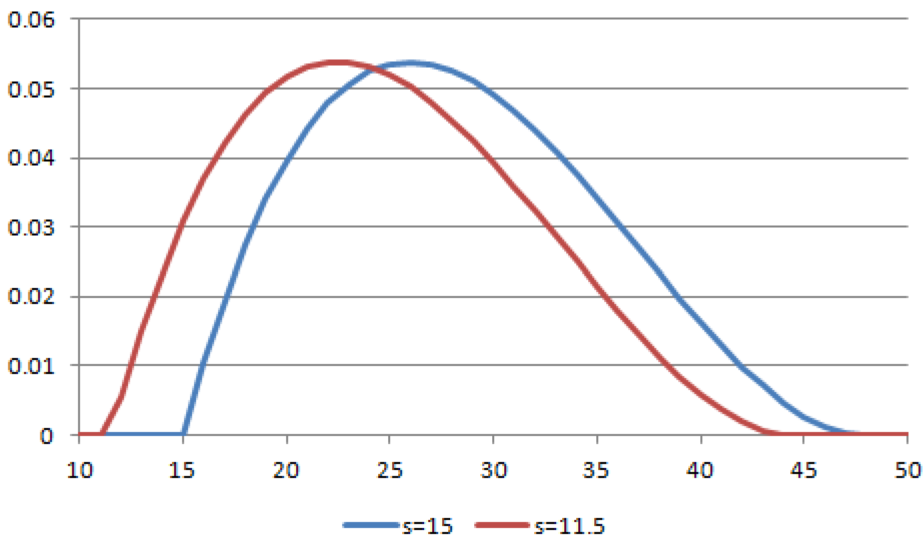
Once the integration is complete, this expression is independent of the value of s. In fact, it can be simplified to the relationship

$$c=\frac{12}{33^4}TFR=\frac{TFR}{98826.75}\approx .00001012\ \ TFR$$

This is a two-parameter model. The constant c is a level parameter, proportional to the TFR, while s is a location parameter, representing the age at which fertility begins. Fertility is assumed to cease 33 years later, at age (s+33). At ages under age s and above age s+33, f(x) is fixed at zero

`In Brass' original exposition of the method, s was allowed to vary between 11.5 and 18.5 to generate a range of fertility schedules, although the effect of changing s merely shifts the curve along the x-axis, and does not alter the underlying age pattern of fertility.

On the graph below, use the arrows to see this variety.



It is a property of this polynomial that the mean age of the fertility schedule is s + 13.2 years, and the maximum fertility rate (that is, the mode) occurs at s + 11 years. These, and many other, properties of the Brass polynomial are described and set out by Retherford (1979).

Romaniuk's three-parameter model

Another class of parametric models of fertility is that proposed by Romaniuk (1973). The model suggested is based on a Pearson Type-1 statistical distribution and takes the form

$$f(x)=T.\left(1+\frac{x}{M-a}\right)^{\left\{\frac{(M-a)(\delta-2)(A-\alpha)}{\delta(A-M)}\right\}}\left(1-\frac{x}{\delta-M+a}\right)^{\left\{\frac{(\delta-M+a)(\delta-2)(A-\alpha)}{\delta(A-M)}\right\}}$$

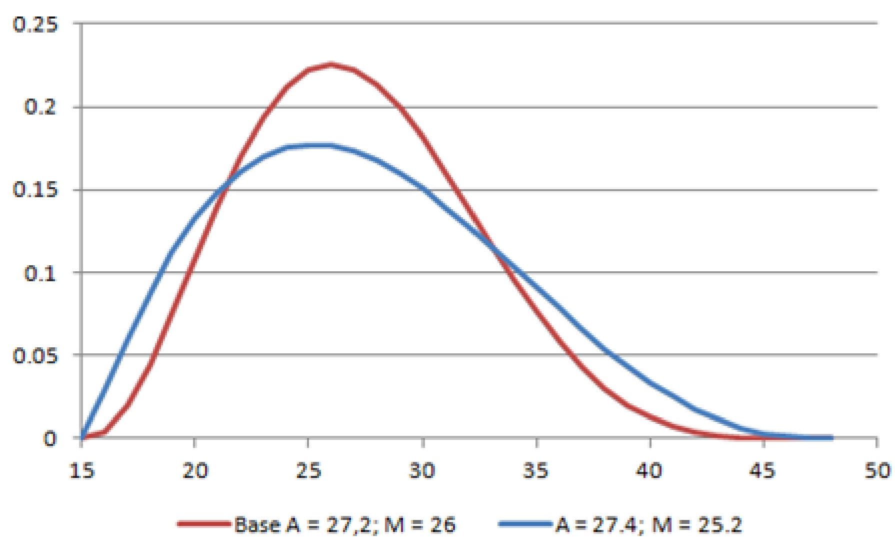
In its most constrained form (that is, taking α and δ as constants) this is a 3-parameter model with parameters T, M and A.

The parameter T is a scaling parameter affecting the level of the fertility distribution. The parameters A and M are the average age and the modal age of the schedule respectively.α and δ represent the start and duration of the fertile age range. If one uses the sameparameters as the Brass polynomial (i.e. setting the starting age, α, to 15; the duration ofreproduction, δ, to 33 years; the modal age, M, to 26 (i.e. 15+11) and the mean age, A, to 28.2 (=15+13.2)), Romaniuk's model has exactly the same form as the original Brass polynomial.

A factor analysis of age-specific fertility schedules would show that three factors are needed to account for about 97% of observed variation in fertility schedules - therefore Romaniuk's model, in its simplified form, has the right number of parameters, but it is not an easy model to handle in practice.

Further, it is not particularly easy to fit, and to obtain good fits in practice thepresumed constants: α and δ, will also need to be varied. The problem is that the location is determined jointly by α, M and A, while the spread is determined jointly by δ, M and A. Thus, the formal parameters and constants between them are not independent in their relationship to the first three principal components of variance in fertility distributions.

Allowing the average age of the model to vary, but keeping the modal age fixed, as in the illustration above, produces fertility distributions with more concentrated and more dispersed shapes than are possible to produce with the Brass polynomial model.



Romaniuk's model is rather better than the simple Brass polynomial at representing the initial years of the fertility distribution, but only if the constant α is treated as a fourth parameter, and allowed to vary. However, with four parameters, the model is somewhat

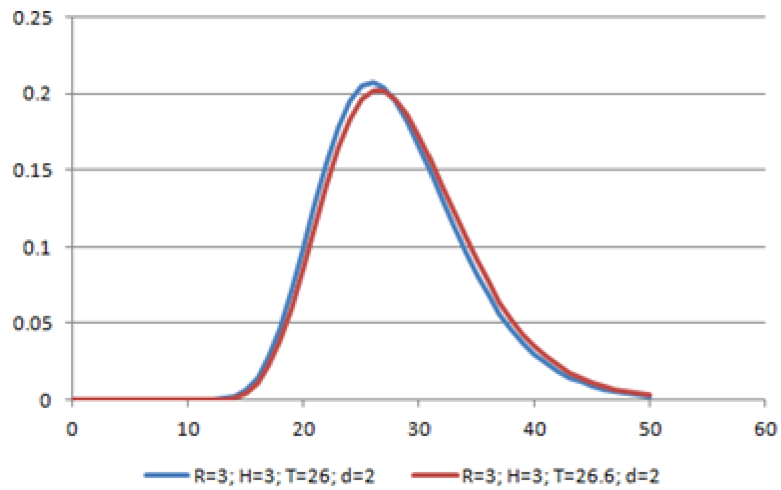
Over specified, especially when working with limited data (e.g. in five-year age groups), or data points whose values are not certain to a high degree of precision.

Hadwiger's model

The final class of parametric models discussed is that proposed by Hadwiger (1940) and refined by Gilje and Yntema (1970). This version of the Hadwiger model is represented by:

$$f(x)= \frac{RH}{T\sqrt{\pi}}\left(\frac{T}{x-d}\right)^{3/2} \exp[-H^2(\frac{T^2+(x-d)^2}{T(x-d)} -2)]$$

This is a four parameter model (with parameters R, H, T and d). Unfortunately, Hadwiger and his successors have used rather different notation compared to the othermodels; so here, for example, T does not relate to the level of fertility, which is accommodated by the scaling factor R.



Population regulation programs in india

India's National Population Policy (NPP) of 2000 aims to achieve a stable population. The NPP's immediate objectives include:

- Addressing unmet needs for contraception, healthcare infrastructure, and health personnel
- Providing integrated service delivery for basic reproductive and child healthcare

The Ministry of Health and Family Welfare is responsible for population control in India. The main strategy is to persuade people to accept the small family norm through advertising and educational efforts.

In 2019, the Population Control Bill was proposed. The bill would have made couples with more than two children ineligible for government jobs and subsidies. The bill was later withdrawn and replaced with the Population (Control, Stabilization and Welfare) Bill in 2021. The National Population Commission is a commission of the Indian government that was established in 2000. The prime minister chairs the commission, and the Deputy Chairman Planning Commission (now NITI Aayog) is the vice chairman.

Population control back in india

A minister in the northeast Indian state of Nagaland last year called on Indians to contribute to a sustainable future by refusing to have children and joining his self-declared "singles movement." "Let us be sensible towards the issues of population growth," Temjen Imna Along tweeted, "or #StaySingle like me." Meanwhile, growing numbers of Indian millennials are deciding against having children for environmental reasons.

Neither development quite amounts to a mass movement. In Indian society, staying childless by choice is still unfathomable for most people. But they are data points of a growing realization among Indians about the problem posed by their country's population, which is now the largest in the world.

The United Nations has stated that by the end of this month, India's population will hit almost 1.43 billion and exceed that of its economic and strategic rival China, long the world's most populous nation. India, however, will have a tougher time than China providing for the bulge, since it is three times smaller in landmass and almost six times behind in GDP.

In 2019, Indian Prime Minister Narendra Modi said that a large population was obstructing India's development. "We have to think if we can do justice to the aspirations of our children," he said. "There is a need to have greater discussion and awareness on population explosion."

The new status has sparked a debate in India about whether it should emulate China's population control policy and create a central law that allows the government to legally, and punitively, enforce a maximum number of children per couple.

India's working-age population comprises 500 million people and is growing, while China's population for the first time is on a decline. Some Indian experts believe these people are India's greatest asset and can fuel India's growth, catapulting it from a developing nation to a developed one. They point to a recent surge in international companies shifting their manufacturing from China to India and suggest that Indians could take jobs in countries with aging populations.

SY Quraishi, a former Indian civil servant and the author of *The Population Myth*, has said that Indians are "the CEOs of the world," presumably in reference to Indian bosses at Google and Microsoft, and that Indians abroad are "the source of an economic revolution that gives back 90 billion dollars in remittances."

Others who hold a more pessimistic view say that while it is true that Indians have done well abroad, a large number are still employed as blue-collar workers, seeking jobs outside India because of a dearth of employment at home.

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