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Unit-IV

Portfolio Theory

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Portfolio Theory

Modern Portfolio Theory (MPT) is a model proposing that investors can construct an optimal portfolio to maximize expected returns for a given level of investment risk.

For example, if you have two investment portfolios A and B with the same level of risk, but portfolio A has a higher expected return, then A is considered more efficient than B. In other words, Portfolio A is delivering more bangs for your investment buck.

Assumptions in MPT include:

- Investors are rational and work to maximize their utility
- Investors act solely on the basis of expected return and variance of return
- All investors have access to the same information
- There are no taxes or transaction costs

For example, MPT assumes that investors are ratio

Behavioural Portfolio Theory (BPT) is an investment theory that integrates elements of psychology and behavioural finance to explain how investors make financial decisions.

For example, some investors are too focused on short-term losses, which may lead them to sell too soon (known as loss aversion), while others may be overly optimistic about their estimates of future returns (overconfidence).

Advantages of Modern Portfolio Theory

Modern Portfolio Theory, developed by Harry Markowitz in 1952, has served as a bedrock for investment principles, holding immense value for both individual and institutional investors. It provides a systematic and robust framework for creating an optimal investment portfolio, primarily benefiting investors in the following aspects:

- **Risk and Return Optimization**: The foremost benefit of Modern Portfolio Theory is its ability to optimise the risk-return trade-off. By strategically diversifying investments, this theory suggests that investors can achieve the highest possible return for a given level of risk—an essential favourable point in risk management.
- **Diversification**: This approach underlines the axiom of "not putting all your eggs in one basket". It promotes diversification, helping investors mitigate risk by spreading investments across a variety of assets that are not perfectly correlated. This diversification can prove particularly beneficial in times of market volatility, as the poor performance of one asset may be counterbalanced by the strong performance of another.

- **Quantifiable measures**: Portfolio Theory goes beyond qualitative assessment to incorporate quantitative analysis. It does so by providing a mathematical approach to portfolio management, employing variance and standard deviation as quantifiable measures of risk. This mathematical basis provides a systematic method for investors to compare and decide on portfolio combinations.
- **Informed Decision Making**: By utilising measures such as the Expected Return on a portfolio, investors can make more informed decisions regarding their investments. This statistical approach allows for effective planning and forecasting, which can aid in more precise financial planning.

Drawbacks and Criticisms: Disadvantages of Portfolio Theory

Despite the manifold strengths of Modern Portfolio Theory, it is subject to certain criticisms and limitations. A comprehensive understanding of these potential downsides aids in better leveraging this theory while avoiding pitfalls. Some notable drawbacks and criticisms include:

- **Based on historical data**: One of the significant limitations of Portfolio Theory is that it is based largely on historical data. It often assumes that past asset performance is an accurate indication of future performance, something that might not always stand true, especially in the highly unpredictable financial markets.
- **Risk is oversimplified**: The theory simplifies risk by associating it largely with volatility. This perception implies that assets with higher price swings are riskier, potentially leaving aside other critical facets of risk, such as liquidity risk, credit risk, and operational risks.
- **Assumptions could be unrealistic**: Certain assumptions upon which the theory is built might be unrealistic; for example, it assumes that investors are rational and avoid risk when they can. In reality, investors might be influenced by behavioural biases and could even seek out risk for potential higher returns.
- **Underestimation of systemic risks**: At times, Portfolio Theory may underestimate systemic risks or "black swan" events that have a significant impact on all asset classes. These risks are not mitigated by diversification, challenging the belief that spreading investments reduces all forms of risk.
- **Limitations with Correlation**: While it’s a fundamental tenet in Portfolio Theory, the idea that asset correlation remains stable over time may not hold in volatile markets, since

correlation trends between assets can change dramatically during periods of financial stress. This might undermine the risk management benefits of diversification offered by Portfolio Theory.
While Portfolio Theory provides an extensive structure for investment decisions, nuances around these limitations should be considered. However, even with its drawbacks, the principles of risk diversification and risk-return optimization hold relevance and continue to be robust strategies in the world of investments.

Risk-return trade-off

Risk-return trade-off meaning describes the investment mind set of investors for the risk exposure included in their investment strategy. The risk and return trade-off states that when investing in equities and mutual funds, the risk exposure and potential profits move in tandem; the higher the risk, the higher the returns. For example, equities offer the highest potential returns for investors but come with the highest level of risk.

Definition: Higher risk is associated with greater probability of higher return and lower risk with a greater probability of smaller return. This trade off which an investor faces between risk and return while considering investment decisions is called the risk return trade off.

Importance of risk-return trade-off in mutual funds

Mutual funds are investment instruments that pool investor money and invest it in various stocks of companies to create a diversified portfolio. They provide investors with different levels of risk and return based on their objectives, risk tolerance, and time horizon, making the risk-return trade-off a vital factor. Here is the importance of the risk-return trade-off in mutual funds.

- Risk management: The trade-off provides a framework to investors for assessing potential risks and rewards for different investment opportunities.
- Return optimisation: Investors can identify investments that offer the best potential return for their level of risk tolerance. This allows them to optimise their portfolio for investment objectives, such as capital preservation, growth, or income.
- Diversification: The risk-return trade-off formula explains the current risk exposure in the investment instruments included in the portfolio. This can allow investors to manage their portfolios and reduce risk by investing in low-risk investment instruments.

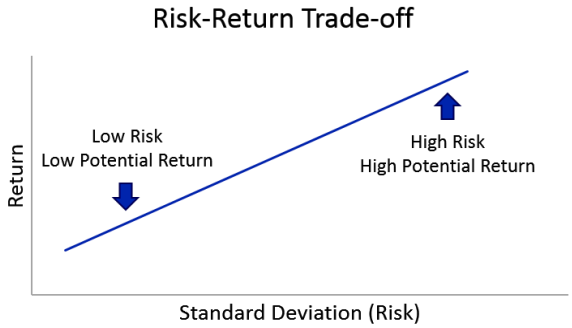
Uses of risk-return trade-off

Risk-return trade-off helps investors manage risk, optimise returns, avoid unnecessary losses, and evaluate the performance of their investments. Here are some of the uses of the risk-return trade-off.

- Measuring singular risk in context: Investors can utilise the trade-off when investing in high-return instruments to ensure they choose those with better return potential. Apart from individual securities, investors can also use the risk-return trade-off in the context of their portfolio as a whole to measure and manage the overall risk.

Investors can choose high-risk, high-return investments such as penny stocks, options, etc., to increase their returns potentially, but also ensure that the high-risk investments do not negatively affect the overall portfolio health.

- Risk-return trade-off at the portfolio level: The risk-return trade-off also exists at the portfolio level. The risk-return trade-off example is when an investor has an all-equity portfolio. Since equities contain the highest risk within all asset classes, the portfolio presents high-profit potential but with a high level of risk. With such a portfolio, the investor can use the trade-off analysis to spread the risk across various sectors or mutual funds. The trade-off assessment can provide valuable insights into the possible achievement of the long-term objectives of the portfolio.



ONE RISKY ASSET

Introduction

A **risky asset** is any investment with uncertain future returns due to market fluctuations, economic conditions, or company-specific risks. The **one risky asset model** focuses on a single asset with variable returns, often compared to a risk-free asset (such as U.S. Treasury bonds). This concept is widely used in finance, portfolio management, and risk analysis.

Key Concepts and Definitions

a. Risky Asset Definition

A risky asset is a financial instrument whose value is volatile and uncertain, meaning it carries a risk of loss along with the potential for gains.

b. Risk-Free Asset Definition

A risk-free asset is an investment with a known return and virtually no risk of loss (e.g., U.S. Treasury bonds).

c. One Risky Asset Model

The one risky asset model assumes an investor can choose between a single risky asset and a risk-free asset to balance risk and return.

Types of One Risky Asset

a. Equities (Stocks)

- Shares in publicly traded companies.
- Returns depend on stock price appreciation and dividends.
- **Example:** Investing solely in Apple (AAPL) stock.

b. Crypto currencies

- Digital assets with high volatility.
- Prices influenced by market sentiment, regulations, and adoption.
- **Example:** Holding only Bit coin (BTC) exposes the investor to crypto risks.

c. Commodities

- Physical goods like oil, gold, or agricultural products.
- Prices influenced by supply and demand, geopolitical events, and economic conditions.
- **Example:** Investing only in crude oil futures.

d. Real Estate

- Properties such as residential, commercial, or industrial real estate.
- Risks include market downturns, interest rates, and economic conditions.
- **Example:** Investing solely in a rental property.

e. Corporate Bonds

- Debt instruments issued by companies.
- Higher risk than government bonds due to potential default.
- **Example:** Buying only Tesla's corporate bonds.

f. Venture Capital or Private Equity

- Investments in startups or private companies.
- High risk with potential for high returns.
- **Example:** Investing only in a single startup.

Methods for Analyzing One Risky Asset

a. Expected Return and Standard Deviation

- **Expected Return (μ):** The average return an investor can expect.
- **Standard Deviation (σ):** Measures volatility and risk.
- **Formula:**

$$E(R) = \sum p_i R_i$$

$$\sigma = \sqrt{\sum p_i (R_i - E(R))^2}$$

b. Capital Asset Pricing Model (CAPM)

- Measures asset risk relative to the market using **Beta (β)**.
- Formula:

$$E(R) = R_f + \beta\,(R_m - R_f)$$

Where:

- R_f = Risk-free rate
- R_m = Market return
- β = Asset's sensitivity to the market

c. Value at Risk (VaR)

- Estimates the potential loss at a given confidence level (e.g., 95%).
- Formula (for normal distribution):

$$VaR = \mu - z\sigma$$

Where z is the z-score for the confidence level.

d. Monte Carlo Simulation

- Runs multiple simulations to predict future asset returns.
- Useful for risk assessment.

e. Sharpe Ratio

- Measures risk-adjusted returns.

- Formula:

$$\text{Sharpe Ratio} = \frac{E(R) - R_f}{\sigma}$$

f. GARCH Model (Generalized Autoregressive Conditional Heteroscedasticity)

- Models time-varying volatility in financial data.
- Helps in forecasting future asset risk.

g. Black-Scholes Model

- Used to price options on risky assets.
- Formula for a European call option:

$$C=S_0N(d_1) – Xe^{-rt}N(d_2)$$

Where N(d) is the cumulative normal distribution function.

Portfolio Optimization with One Risky Asset

- If an investor holds **one risky asset** and a **risk-free asset**, the optimal allocation depends on risk tolerance.
- **Risk-Loving Investor** → holds more of the risky asset.
- **Risk-Averse Investor** → allocates more to the risk-free asset.
- **Optimal Portfolio Formula:**

$$w = \frac{E(R) - R_f}{A\sigma^2}$$

Where:

- w = Weight in risky asset
- A = Investor's risk aversion

Examples of One Risky Asset Investments

a. Stock Investment Example

- Investor buys only **Tesla (TSLA)** stock.
- Risks: Tesla's financial health, market conditions, competition.
- Rewards: Stock price appreciation, dividends.

b. Crypto currency Example

- Investor holds **Ethereum (ETH)** only.
- Risks: Volatility, regulatory crackdowns.
- Rewards: High potential returns from adoption.

c. Commodity Example

- Investor trades only **gold futures**.
- Risks: Geopolitical risks, inflation effects.
- Rewards: Safe-haven asset during crises.

d. Real Estate Example

- Investor owns one **rental property**.
- Risks: Property market downturns, tenant defaults.
- Rewards: Rental income, property appreciation.

Risk Management Strategies for One Risky Asset

a. Hedging

- Using **derivatives (options, futures)** to reduce risk.
- Example: Buying a **put option** on a stock investment.

b. Diversification (if allowed)

- Holding a **mix of assets** to reduce risk.
- Example: Combining stocks and bonds.

c. Stop-Loss Orders

- Setting a **predefined exit price** to minimize losses.

d. Asset Allocation

- Adjusting **portfolio weights** between risky and risk-free assets.

Conclusion

The **one risky asset model** helps investors analyze and manage risk when investing in a single asset. While potential rewards can be high, so are the risks. Using statistical models, risk management techniques, and financial theories like CAPM and VaR can help in making informed decisions.

Risk – Free Asset:

A **risk-free asset** is an investment with a guaranteed return and no risk of financial loss. In reality, no asset is completely risk-free, but certain assets come very close, primarily due to government backing and liquidity. The most common example of a risk-free asset is a **U.S. Treasury Bill (T-Bill)**. Here's a detailed breakdown:

U.S. Treasury Bill (T-Bill) – The Classic Risk-Free Asset

1. Issuer:

- Issued by the **U.S. Department of the Treasury**, backed by the U.S. government.

2. Maturity Periods:

- T-Bills come in different maturities, typically **4, 8, 13, 26, and 52 weeks**.
- They are **short-term** instr uments, making them highly liquid.

3. Returns & Interest Payments:

- T-Bills are sold at a **discount** to face value.
- They do **not** pay periodic interest (zero-coupon bonds).
- Instead, investors earn a return when the bill matures at its full face value.
- Example: You buy a $1,000 T-Bill for $980. At maturity, you receive $1,000, earning a $20 return.

4. Risk Level:

- Virtually **zero default risk** because the U.S. government is highly unlikely to default.
- No **credit risk**, unlike corporate bonds or other investments.
- However, **inflation risk** and **interest rate risk** can still impact real returns.

5. Market & Liquidity:

- Highly liquid actively traded in secondary markets.
- Easily bought and sold through the **Treasury Direct** website, brokers, and financial institutions.

6. Currency Risk:

- For U.S. investors, no currency risk.
- For foreign investors, returns depend on exchange rate fluctuations between their currency and the U.S. dollar.

7. Use in Finance & Investment Models:

- Considered the "risk-free rate" in financial models like **CAPM (Capital Asset Pricing Model)**.
- Used as a benchmark for comparing returns on risky assets.

T-Bills Considered Risk-Free

1. **Government Guarantee:** The U.S. government has never defaulted on T-Bills.
2. **Short Maturity:** The short-term nature reduces exposure to economic fluctuations.

3. **High Liquidity:** Easy to convert into cash without significant loss.

Portfolio of Two Assets

A "portfolio of two assets" in financial statistics refers to an investment strategy where an investor holds only two different assets within their portfolio, allowing them to analyze the risk and return characteristics based on the interaction between those two assets, primarily considering their individual variances and the covariance (correlation) between them.

Calculating Portfolio Return:

The expected return of a two-asset portfolio is a weighted average of the expected returns of each individual asset, where the weights represent the proportion of the portfolio invested in each asset.

Portfolio Variance:

The most important aspect of a two-asset portfolio is calculating its variance, which takes into account not just the individual variances of each asset but also the covariance between them.

Covariance and Correlation:

Covariance measures how two assets move together, while correlation is a standardized version of covariance that indicates the direction and strength of that relationship.

Diversification Benefit:

By combining assets with a negative correlation, an investor can potentially reduce the overall risk of their portfolio even if the individual assets are considered risky.

A portfolio of these two assets is characterized by the value invested in each asset.
Let V1 and V2 be the dollar amount invested in asset 1 and 2, respectively. The total value of the portfolio is $V = V_1 + V_2$.

Consider a portfolio in which

- $w_1 = V_1/V$ is the weight on asset 1
- $w_2 = V_2/V$ is the weight on asset 2.

Then, $w_1 + w_2 = 1$.

Expected return on a portfolio with two assets

Expected portfolio return:

$$\bar{r}_p = w_1 \bar{r}_1 + w_2 \bar{r}_2 .$$

Unexpected portfolio return:

$$\tilde{r}_p - \bar{r}_p = w_1(\tilde{r}_1 - \bar{r}_1) + w_2(\tilde{r}_2 - \bar{r}_2).$$

Variance of return on a portfolio with two assets

The variance of the portfolio return:

$$\sigma_p^2 = \mathrm{Var}[\tilde{r}_p] = \mathrm{E}[(\tilde{r}_p - \bar{r}_p)^2]$$

$$= w_1^2 \sigma_1^2 + w_1^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12} .$$

Variance of the portfolio is the sum of all entries of the following table

	$w_1 \tilde{r}_1$	$w_2 \tilde{r}_2$
$w_1 \tilde{r}_1$	$w_1^2 \sigma_1^2$	$w_1 w_2 \sigma_{12}$
$w_2 \tilde{r}_2$	$w_1 w_2 \sigma_{12}$	$w_2^2 \sigma_2^2$

Utility Theory:

In finance, "utility theory" refers to the economic concept that individuals make investment decisions based on maximizing their personal "utility" or satisfaction, which is essentially the subjective value they place on potential outcomes, rather than solely focusing on the highest possible monetary return; it helps explain why investors might choose a less risky option even if it offers a lower expected return, depending on their individual risk tolerance and preferences as represented by their "utility function."

Key points about utility theory in finance:

Utility function:

A mathematical formula that represents an individual's level of satisfaction (utility) associated with different levels of wealth or investment outcomes.

Risk aversion:

Most individuals are considered "risk averse," meaning they prefer a certain, lower return to a potentially higher return with greater risk, which is reflected in a concave utility function.

Expected utility:

The weighted average of the utility of each possible outcome in a risky investment, where the weights are the probabilities of those outcomes occurring.

utility theory is applied in finance:

Portfolio optimization:

Investors use their utility function to determine the optimal mix of assets in their portfolio, balancing risk and potential return to maximize their expected utility.

Insurance decisions:

Understanding utility helps explain why people buy insurance, as the potential utility gained from risk protection outweighs the cost of premiums, even if the expected payout is lower than the premium.

Important considerations:

Subjective nature:

Utility is a subjective concept, meaning different individuals will have different utility functions based on their personal preferences and circumstances.

Limitations:

Real-world decision making is often complex and influenced by factors beyond pure utility calculations, such as emotions and cognitive biases.

Applications of utility theory

Portfolio optimization: Using utility functions to select investment portfolios that balance risk and return based on an investor's risk tolerance.

Insurance decisions: Explaining why individuals purchase insurance even when the expected payout is less than the premium, as the utility gained from risk reduction outweighs the cost.

Important concepts related to utility theory:

Marginal utility:

The additional satisfaction gained from consuming one more unit of a good, which usually diminishes as more units are consumed.

Indifference curve:

A graphical representation of combinations of two goods that provide an individual with the same level of utility.

Expected utility theory:

The application of utility theory to uncertain situations, calculating the expected utility of a choice by weighting the utility of each possible outcome by its probability.