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Unit-II

Risk and Return

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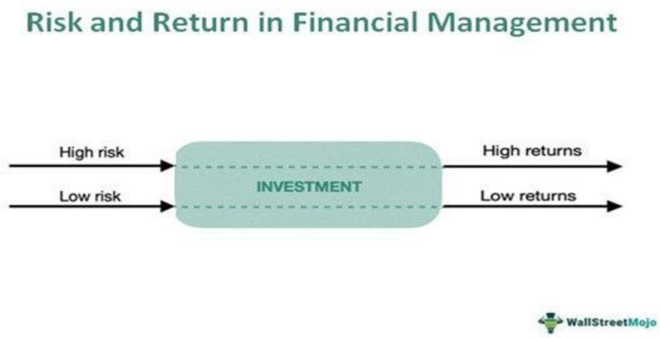
RISK AND RETURN

Risk and return in financial management is the risk associated with a certain investment and its returns. Usually, high-risk investments yield better financial returns, and low-risk investments yield lower returns. That is, the risk of a particular investment is directly related to the returns earned from it.



Types of Risk

In investing, risk and return are highly correlated. Increased potential returns on investment usually go hand-in-hand with increased risk. Different types of risks include project-specific risk, industry-specific risk, competitive risk, international risk, and market risk. Return refers to either gains or losses made from trading a security.

- **Market risk** – It is also called systematic risk and arise due to various market related factors like economic and political problems, interest rate and currency fluctuations, etc. They have a huge impact on the investors.
- **Specific risks** – They are related mostly to company itself. They may be controlled through diversification an monitoring.
- **Credit risk** – This is related to credit worthiness of the company or business. If the financial condition of the business is good, it will be able to meet its current and future obligations and repay its debts on time. This will lead to good credit rating. Credit risk is the result of deteriorating financial health of the company.
- **Liquidity risk** – This is the result of the business not being able to earn good revenue to meet its financial obligations and maintain high working capital.
- **Interest rate risk** – The fluctuations in the interest rates in an economy can affect the business's borrowing capacity.
- **Inflation –** The inflation leads to erosion of value of investments and the value of cash flows in future.

Systematic risk

In investment decisions, both Systematic and Un-systematic risks play important role. Systematic risk includes: (a) market risk, (b) interest rate risk, (c) inflation rate risk (purchasing power risk).

market risk

The term market risk, also known as systematic risk, refers to the uncertainty associated with any investment decision. Price volatility often arises due to unanticipated fluctuations in factors that commonly affect the entire financial market.

Interest rate risk

Interest rate risk arises from unanticipated fluctuations in the interest rates due to monetary policy measures undertaken by the central bank. The yields offered on securities across all markets must get equalized in the long run by adjustment of market demand and supply of the instrument. Hence, an increase in the rates would cause a fall in the security price. It is primarily associated with fixed-income securities.

Inflation risk

Inflation risk means reduced purchasing power. The same amount of money that could buy you things today will buy you fewer things in the future as prices increase. Hence, it is also known as purchasing power risk. It results in the erosion of money in an economy over time.

Unsystematic Risk

Unsystematic Risk are, generally, indicated by leverages of a firm. Unsystematic risk includes the following types of risk: (a) Business risk, and (b) Financial risk.

Business risk

Business risk is the exposure a company or organization has to factor(s) that will lower its profits or lead it to fail. Anything that threatens a company's ability to achieve its financial goals is considered a business risk. There are many factors that can converge to create business risk.

Operating Risk = Degree of Operating Leverage

Degree of Operating Leverage = Percentage change in operating income / Percentage change in sales

Financial risk

Financial risk is the possibility of losing money on an investment or a business venture. Some more common and distinct financial risks include credit risk, liquidity risk, and operational risk.

Financial Risk = Degree of Financial Leverage

Degree of Financial Leverage = Percentage change in EPS / Percentage change in EBIT

Return

Return is a profit on an investment. It comprises any change in value of the investment, and/or cash flows (or securities, or other investments) which the investor receives from that investment over a specified time period, such as interest payments, coupons, cash dividends and stock dividends.

$$\text{Total Return} = \text{Income} + \text{Change in Price of Investment } (+/-)$$

Return on equity (in percent value) will be computed as given below:

$$\text{Return} = \frac{\text{Income}}{\text{Price paid for security}} + \frac{\text{Change in price of a security over a period}}{\text{Price paid for security}}$$

$$\text{Or,}\quad \text{Return (\%)} = \frac{\text{Income + Change in price of security over a period}}{\text{Price paid for security}} \times 100$$

$$\text{Or,}\quad \text{Return} = \frac{D_1}{P_0} + \frac{(P_1 - P_0)}{P_0}$$

$$\text{Or,}\quad \text{Return (\%)} = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100$$

Where,

R(%) = Rate of return, i.e., yield.

D_1 = Dividend received at the end of year, denoted by 1.

Po = The value of investment made in year '0'.

P_1 = The value of investment in year '1', to be considered as the year in which security will be realised/sold.

Similarly, the yield on fixed interest-bearing securities may also be computed by replacing 'D' with 'I' i.e., interest.

$$\text{Return (\%) on debt security} = \frac{I_1 + (P_1 - P_0)}{P_0}$$

Types of Return:

The return on an investment is expressed as a percentage and considered a random variable that takes any value within a given range. Several factors influence the type of returns that investors can expect from trading in the markets.

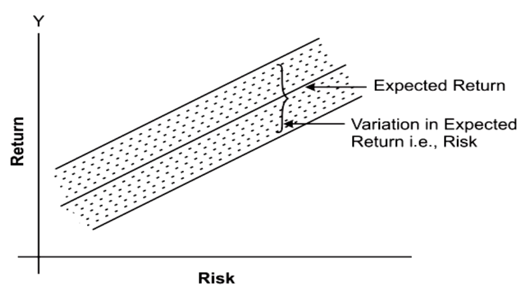
- **Capital gains** – Any good investment will rise in value as time passes by. Thus the assets will be valued higher if they are sold later on as compared to its purchase price, giving capital gain.

- **Dividends** – they are a steady source of income for investors who invest in shares of companies giving regular dividends which are a part of the profits set aside for investors.
- **Interest** – Borrowers like individuals or corporates borrow money for meeting expenses or capital requirements. The lenders give the funds to get interest on the principal amount which is a return on investment for the lenders.
- **Rental income** – Any property rented out can earn rent on a regular basis, which is also a return in the real estate property.
- **Return from currency trading** – Profits earned from trading in exchange rates by using the differences is exchange rate of different currency is also a form of return for those who do currency trading.

Thus the above a some important types of **risk and return on investment** that are very popular in the financial market.

Measurement of Risk

Measurement of risk is as essential as measurement of return on the investment. Risk is a variation in the expected return. Thus, it is incorrect to estimate return on investment in the absence of risk associated with it. Since it indicates variation in expected return, therefore, the statistical techniques of dispersions may be used to estimate risk on securities.



Risk and Return (Variation in Return)

(i) Standard Deviation

Standard Deviation measures the variation in actual return from the expected average return. A low value of standard deviation indicates actual return likely to be close to average return, on the other hand, a high value of standard deviation shows lesser possibility of actual return close to average return. The standard deviation (SD) is symbolized with sigma, 'σ'. The statistical formula to calculate standard deviation is:

$$SD = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \bar{X}_I)^2}{N}}$$

Where,

X_i = Actual return on investment,

$\bar{X}_I$ = Average return, and

N = Number of observations

Illustration: The expected returns of last five years are provided below. Compute expected risk on these returns.

Year	Return
1	50
2	70
3	80
4	100
5	90

Solution:

Year	Return (Xi)	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$
1	50	–28	784
2	70	–8	64
3	80	2	4
4	100	22	484
5	90	12	144
	Mean = 78		1480

$$SD = \sqrt{\frac{1480}{10}} = 17.20\%$$

(ii) Measurement of Systematic Risk

The systematic risk of a security is measured by a statistical measure called beta. The value of the beta may be computed from the historical data. To compute beta, there are two methods in statistics, first, correlation method, second, regression method. Under the correlation method, beta is computed with following formula:

$$\beta = \frac{r_{im}\sigma_i\sigma_m}{\sigma_m^2}$$

ẞ = Beta, or measurement of systematic risk.

r_{im} = Correlation coefficient of returns of stock and returns of the market index.

σ_i = Standard deviation of return of stock i.

σ_m = Standard deviation of returns of the market index.

σ^2_m = Variance of the market returns.

(iii) Regression Method

The regression method is widely used method to compute beta. The beta shows a linear relationship between dependent and independent variable. There are two values computed under regression model viz., alpha (a) and beta (B). The value of alpha remains constant, and the value of beta makes change in expected return. The regression equation is written in the following form:

$$Y = \alpha + \beta X_i$$

Where,

Y = Dependent variable, i.e., expected return

X_i = Independent variables

'α' = Alpha constant

'β' = Beta constant

This equation is known as characteristic line.

The value of alpha and beta is computed as shown below.

$$\alpha = \overline{Y} - \beta Xi$$

$$\beta = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - \Sigma(X)^2}$$

Where,

N = Number of observations

Y hat = Mean value of the dependent variable

X hat = Mean value of the independent variable

Y = Dependent variable

X = Independent variable

The same formula may be applied in stock market return, by rewriting equation of characteristic line as given below.

$$R_i = \alpha + \beta R_m$$

Where,

R_i = Return on the individual security

R_m = Return on the market index

'α' = Estimated return of the security when the market is stationary, or a constant,

'β' = Change in the return of the individual security in response to change in the return of the market index.

Illustration The return on individual security (R_i) and market return (R_m) is given below.

R_i	14	18	6	12	13	14	11	6	9	8
R_m	16	20	9	8	10	9	11	18	17	15

Compute alpha and beta from the above.

R_i	R_m	$R_i * R_m$	R_i^2
14	16	224	196
18	20	360	324
6	9	54	36
12	8	96	144
13	10	130	169
14	9	126	196
11	11	121	121
6	18	108	36
9	17	153	81
8	15	120	64
$\Sigma R_i = 111$	**$\Sigma R_m = 133$**	**$\Sigma R_i * R_m = 1492$**	**$\Sigma R_i^2 = 1367$**
Mean (R_i) 11.1	**Mean (R_m) = 13.3**		

$$\beta = \frac{n\Sigma R_i * R_m - (\Sigma R_i)(\Sigma R_m)}{n\Sigma R_i^2 - \Sigma(R_i)^2}$$

$$\beta = \frac{10 * 1492 - 111 * 113}{10 * 1367 - (111)^2}$$

$$\beta = 0.1163$$

$$\alpha = 11.1 - (0.1163 * 13.3)$$

$$\alpha = 12.00$$

From the above, the value of beta is 0.1163 which is less than 1 so it can be concluded that investment in the security is less aggressive.

Now this way, total risk on a security may be computed. Moreover, the total variance can be explained by adding systematic variance as well as unsystematic variance.

Adjusting for dividends

If an asset pays a dividend, D_1, sometime between months t-1 and t, the total net return calculation becomes

$$R_t^{total} = P_t + D_t – P_{t-1} / P_{t-1} = P_t - P_{t-1} / P_{t-1} + D_t / P_{t-1}$$

where $P_t - P_{t-1} / P_{t-1}$ is referred as the capital gain and D_t / P_{t-1} dividend yield. The total gross return is

$$1 + R_t^{total} = P_t + D_t – P_{t-1}$$

The formula for computing multi period return remains the same except that one-period gross returns are computed using

Gross Rate of Return

The gross rate of return is the total rate of return on an investment before the deduction of any fees, commissions, or expenses. The gross rate of return is quoted over a specific period of time, such as a month, quarter, or year. This can be contrasted with the net rate of return, which deducts fees and costs to provide a more realistic measurement of return.

$$\text{Gross rate of return} = \frac{(\text{Final value} - \text{initial value})}{\text{Initial value}}$$

Continuously Compounded Returns

In this section we define continuously compounded returns from simple re-turns, and describe their properties.

One-period Returns

Let R_t denote the simple monthly return on an investment. The continuously compounded monthly return, r_t, is defined as:

$$r_t = \ln(1+R_t) = \text{In } (P_t / P_{t-1})$$

$$e^{rt} = 1 + R_t = P_t / P_{t-1}$$

Rearranging we get

$$P_t = P_{t-1}e^{rt},$$

so that r_t is the continuously compounded growth rate in prices between months t-1 and t. This is to be contrasted with R_t, which is the simple growth rate in prices between months t-1 and t without any compounding. Furthermore, since In (x/y) = ln(x) - ln(y) it follows that

$$\begin{aligned} r_t &= \ln\left(\frac{P_t}{P_{t-1}}\right) \\ &= \ln(P_t) - \ln(P_{t-1}) \\ &= p_t - p_{t-1}, \end{aligned}$$

where p_t = In(P_t). Hence, the continuously compounded monthly return, r_t, can be computed simply by taking the first difference of the natural logarithms of monthly prices.

Multi-Period Returns

The relationship between multi-period continuously compounded returns and one-period continuously compounded returns is more simple than the relationship between multi-period simple returns and one-period simple returns.

To illustrate, consider the two-month continuously compounded return de- fined as:

$$r_t(2) = \ln(1 + R_t(2)) = \ln\left(\frac{P_t}{P_{t-2}}\right) = p_t - p_{t-2}.$$

Taking exponentials of both sides shows that

$$P_t = P_{t-2}e^{r_t(2)}$$

so that $r_t(2)$ is the continuously compounded growth rate of prices between months t - 2 and t. Using

$$\frac{P_t}{P_{t-2}} = \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}}.$$

and the fact that In(x.y) = In(x) + In(y) it flows that

$$\begin{aligned} r_t(2) &= \ln\left(\frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}}\right) \\ &= \ln\left(\frac{P_t}{P_{t-1}}\right) + \ln\left(\frac{P_{t-1}}{P_{t-2}}\right) \\ &= r_t + r_{t-1}. \end{aligned}$$

Hence the continuously compounded two-month return is just the sum of the two continuously compounded one-month returns. Recall, with simple returns the two-month return is a multiplicative (geometric) sum of two one-month returns.

Portfolio Returns

The continuously compounded portfolio return is defined by

$$r_t = \ln(1 + R_t) = \ln\left(\frac{P_t}{P_{t-1}}\right),$$

,

where R_t is computed using the portfolio return. However, notice that

$$r_{p,t} = \ln(1 + R_{p,t}) = \ln(1 + \sum_{i=1}^{n} x_i R_{i,t}) \neq \sum_{i=1}^{n} x_i r_{i,t},$$

where $r_{i,t}$ denotes the continuously compounded one-period return on asset i. If the portfolio return

$$R_{p,t} = \sum_{i=1}^{n} x_i R_{i,t}$$

is not too large then $r_{p,t} \approx R_{p,t}$ otherwise, $R_{p.t} > r_{p.t}$.

Net return

Net return is the profit a company or individual makes on an investment after taxes, deductions, depreciation, and other expenses have been subtracted. When it comes to investing, it is the capital you receive in your bank account.

A rate of return (RoR)

A rate of return (RoR) is the net gain or loss of an investment over a specified time period, expressed as a percentage of the investment's initial cost. When calculating the rate of return, you are determining the percentage change from the beginning of the period until the end.

$$\text{Rate of return} = [\frac{(\text{Current value} - \text{Initial value})}{\text{Initial value}}] \times 100$$

The internal rate of return (IRR)

The internal rate of return (IRR) is the annual rate of growth that an investment is expected to generate. IRR is calculated using the same concept as net present value (NPV), except it sets the NPV equal to zero.

Discounted cash flow (DCF)

Discounted cash flow (DCF) is a valuation method that estimates the value of an investment using its expected future cash flows. Analysts use DCF to determine the value of an investment today, based on projections of how much money that investment will generate in the future.

$$IRR = NPV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} - C_0 = 0$$

where:

T = total number of time periods

t = time period

C_t = net cash inflow-outflows during a single period t

C_0 = baseline cash inflow-outflows

r = discount rate

Log returns

Log returns, also known as logarithmic returns or continuously compounded returns, are a way to measure the percentage change in an asset's value over time. They are calculated using the natural logarithm of the ratio of the asset's current price to its previous price.

$$L_t = \frac{P_{t+1}}{P_t} - 1$$

$$where: L_t = Linear\ return\ at\ time\ t$$
$$P_t = Price\ at\ time\ t$$

Random walk

Random walk theory refers to the idea that changes in asset prices are random and stock prices move unpredictably. Random walk theory suggests that changes in asset prices are random. This means that stock prices move unpredictably, so that past prices cannot be used to accurately predict future prices.

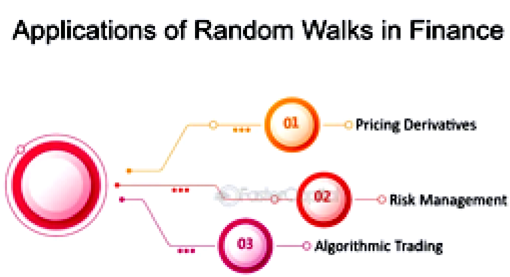
Applications of Random Walks

Applications of Random Walks in Finance:

Random walks are not only a fascinating mathematical concept, but they have important applications in different fields, especially in finance. Random walks can be used to model the price fluctuations of financial assets, such as stocks, bonds, and commodities. Financial analysts and traders use random walk models to make predictions about future asset prices and to understand the risk associated with different investment strategies. From a theoretical perspective, random walk models are used to test the efficiency of financial markets and to develop trading algorithms that can exploit market inefficiencies. In this section we will explore the applications of random walks in finance in more detail.

- **Pricing Derivatives**: One of the main applications of random walks in finance is to price financial derivatives, such as options and futures contracts. derivatives are financial instruments whose value is derived from the underlying asset, such as a stock or a commodity. Random walk models are used to simulate the behavior of the underlying asset and to estimate the expected value of the derivative at maturity. For example, the black- Scholes model uses a random walk to model the behavior of the underlying asset and to derive the theoretical value of a European call or put option.
- **Risk Management:** Random walks are also used to manage risk in financial markets. risk management is the process of identifying, assessing, and controlling risks that arise from financial transactions. Random walk models are used to estimate the volatility of financial assets, which is a measure of the degree of variation of the asset's price over time. Volatility is an important input in risk management models, such as Value- at-Risk (VaR), which is a statistical measure of the maximum expected loss of a portfolio over a given period of time.
- **Algorithmic Trading:** Random walks are also used to develop trading algorithms that can exploit market inefficiencies. Algorithmic trading is the use of computer programs to execute trades in financial markets. Random walk models are used to identify patterns in asset prices that can be used to predict future price movements. For example, mean reversion strategies use random walk models to identify assets that are overvalued or undervalued relative to their historical averages. These strategies buy assets that are undervalued and sell assets that are overvalued, in order to profit from the reversion of prices to their historical means.

Random walks play a crucial role in understanding the behaviour of financial markets. They are used to price derivatives, manage risk, and develop trading strategies. Random walks are not only a theoretical concept, but they have practical applications that are used by financial analysts and traders on a daily basis.



Origins of the random walk hypothesis

Efficient Market

A market where there are large numbers of rational profit maximizes actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants.

Efficient Market Hypothesis

- ❖ To test the form of market - extent of efficiency.
- ❖ To make sure that one can accurately forecast the market, discover the market trend and help investors to make critical decisions.

Types of efficient market hypothesis (EMH)

The Efficient Markets Hypothesis (EMH) consists of three progressively stronger forms:

- ❖ Weak Form
- ❖ Semi-strong Form
- ❖ Strong Form.

The weak form

- ❖ The weak form of the EMH says that past prices, volume, and other market statistics provide no information that can be used to predict future prices.
- ❖ Price changes should be random because it is information that drives these changes, and information arrives randomly.
- ❖ Prices should change very quickly and to the correct level when new information arrives.
- ❖ Most research supports the notion that the markets are weak form efficient.

The semi-strong form

- ❖ The semi-strong form says that prices fully reflect all publicly available information and expectations about the future.
- ❖ This suggests that prices adjust very rapidly to new information, and that old information cannot be used to earn superior returns.
- ❖ The semi-strong form, if correct, repudiates fundamental analysis.
- ❖ Most studies find that the markets are reasonably efficient in this sense, but the evidence is somewhat mixed.

The strong form

- ❖ The strong form says that prices fully reflect all information, whether publicly available or not.
- ❖ Even the knowledge of material, non- public information cannot be used to earn superior results.
- ❖ Most studies have found that the markets are not efficient in this sense.

Six lessons for market efficiency

1. Markets Have No Memory.
2. Trust Market Prices.
3. Read The Entrails.
4. There Are No Financial Illusions.
5. Do-It-Yourself Alternative.
6. Seen One Stock, Seen Them All.

Discrete compounding and Continuous compounding

Discrete Compounding

Discrete compounding refers to the method by which interest is calculated and added to the principal at certain set points in time. For example, interest may be compounded weekly, monthly, or yearly.

Discrete compounding can be compared with continuous compounding, which uses a formula to compute interest as if it were being constantly calculated and added to the principal amount.

- Discrete compounding refers to payments made on balances at regular intervals such as weekly, monthly, or yearly.

- Continuous compounding yields the largest net return and computes (using calculus) interest paid hypothetically at every moment in time.

Discrete Compounding

If the interest rate is simple (no compounding takes place), then the future value of any investment can be written as:

$$FV = P(1 + \frac{r}{m})^{mt}$$

where:

FV Future value

P = Principal

(r/m) = Interest rate

mt = Time period

Compounding interest calculates interest on the principal and accrued interest. When interest is compounded discretely, its formula is:

$$\mathbf{FV} = \mathbf{P}(1 + \frac{r}{m})^{mt}$$

where:

t = The term of the contract (in years)

m = The number of compounding periods per year

Continuous Compounding

Continuous compounding introduces the concept of the natural logarithm. This is the constant rate of growth for all naturally growing processes. It's a figure that developed out of physics.

The natural log is typically represented by the letter e. To calculate continuous compounding for an interest-generating contract, the formula needs to be written as:

$$FV = P \times e^{rt}$$

Importance of continuous compounding

Here are some reasons that make continuous compounding an essential component of financial planning:

1. Earning potential: Continuous compounding helps you determine the potential yield you can earn as interest accrues over time. It gives you a clearer picture of how your investment could grow steadily.

2. Expectations: It helps you anticipate the returns you can expect from your investment when subject to continuous compounding. This helps in setting realistic financial goals.

3. Strategic reinvestment: With continuous compounding, the interest keeps building on itself. This, in turn, gives your money a higher base for subsequent interest calculations. This helps you decide where to reinvest your earned interest and aim for enhanced profits.

4. Faster growth: Continuous compounding outpaces simple interest calculations, considering both the initial investment and the accumulated interest. This accelerates the growth of your investment over time.

5. Accelerated effect: Through continuous compounding, your money multiplies accelerated. The more frequent the compounding periods, the more significant the impact of compound interest on your investment.

6. Long-term advantage: Continuous compounding is particularly beneficial for long-term investments. Over time, the compounding effect becomes more pronounced, boosting your investment returns significantly.