BHARATHIDASAN UNIVERSITY

CENTRE FOR DIFFERENTLY ABLED PERSONS

TIRUCHIRAPPALLI - 620024.

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- Unit : Unit II
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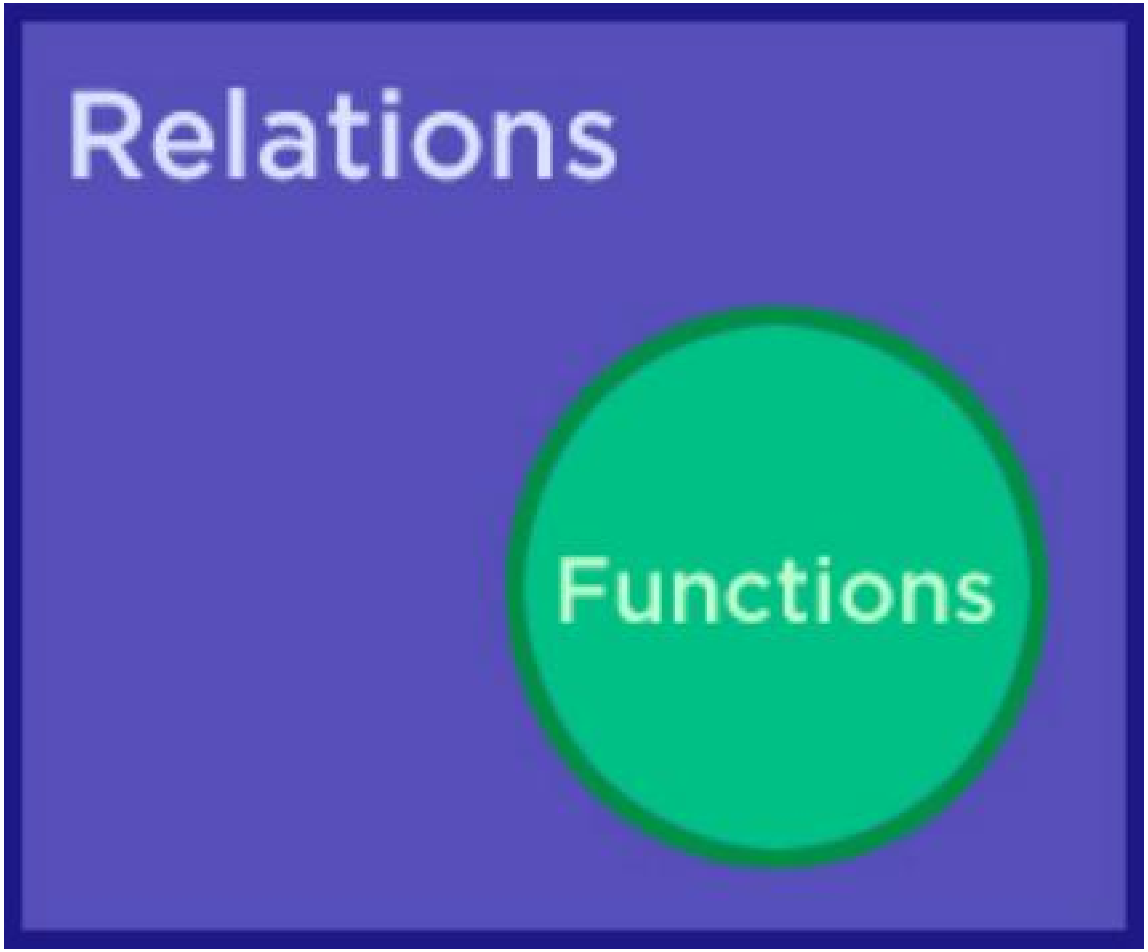

Relations
Functions

FUNCTION

- A function is a binary relation between two sets that associates every element of the first set to exactly one element of the second set.

- Functions are generally represented as f(x)f(x)

- Let, f(x)= x^{3}f(x)=x^{3}

- It is said as f of x is equal to x cube.

- Functions can also be represented by g(), t(),… etc.

Example 1:

Find the output of the function $g(t)= 6t^2 +5$ at

(i) $t = 0$

(ii) $t = 2$

Solution:

The given **function $g(t)= 6t^2 +5$**

(i) At $t = 0$, $g(0)= 6(0)^2 +5$

$= 5$

(ii) At $t = 2$, $g(2)= 6(2)^2 +5$

$= 6(4) + 5$

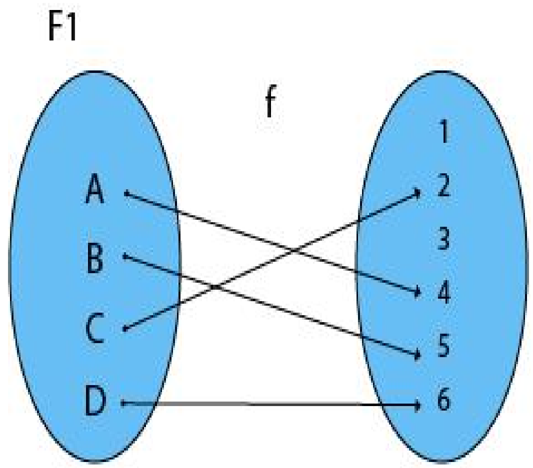
$= 24 + 5$

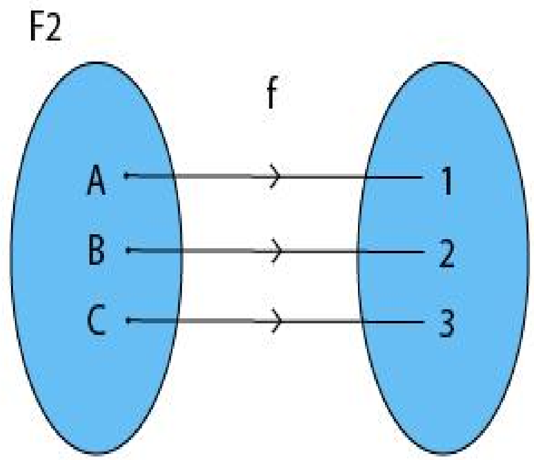
$= 29$

1. One to One Function

- A function in which one element of Domain Set is connected to one element of Co-Domain Set.

- A function f from A to B is called one-to-one (or 1-1) if whenever f (a) = f (b) then a = b.



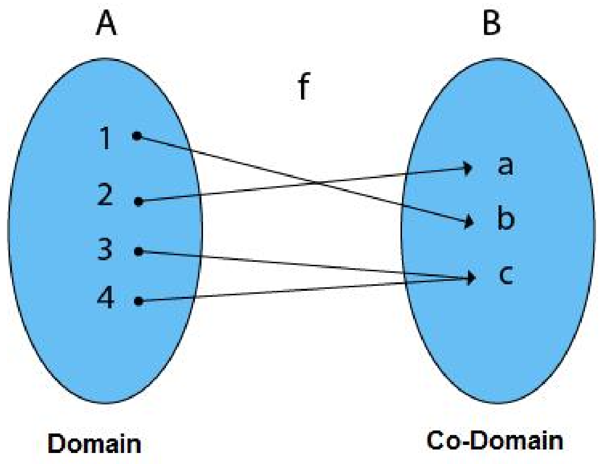


2. Surjective (Onto) Functions:

- A function in which every element of Co-Domain Set has one pre-image.

Example:

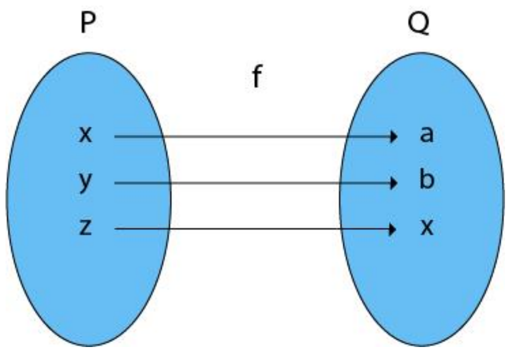
- Consider, A = {1, 2, 3, 4}, B = {a, b, c} and f = {(1, b), (2, a), (3, c), (4, c)}.
- It is a Surjective Function, as every element of B is the image of some A



- In an Onto Function, Range is equal to Co-Domain.

3. Bijective (One-to-One Onto) Functions:

- A function which is both injective (one to - one) and surjective (onto) is called bijective (One-to-One Onto) Function.



Consider P = {x, y, z}

Q = {a, b, c}

and f: P → Q such that

f = {(x, a), (y, b), (z, c)}

The f is a one-to-one function and also it is onto.

So, it is a bijective function.

4. Into Functions:

- A function in which there must be an element of co-domain Y does not have a pre-image in domain X.

Example:

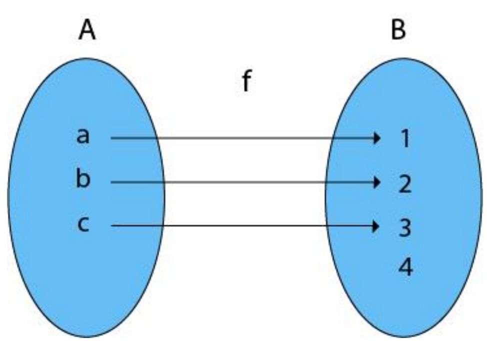
Consider, A = {a, b, c}

B = {1, 2, 3, 4}   and f: A → B such that

f = {(a, 1), (b, 2), (c, 3)}

In the function f, the range i.e., {1, 2, 3} ≠ co-domain of Y i.e., {1, 2, 3, 4}

Therefore, it is an into function



5. Many-One Functions:

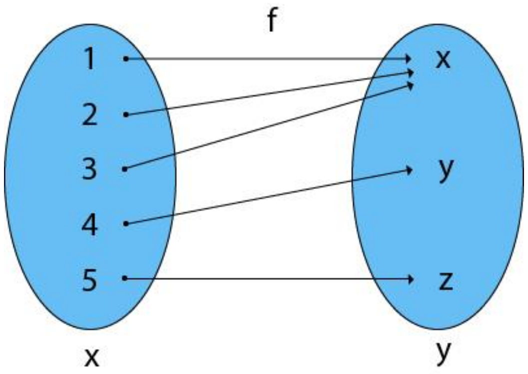
- Let f: X → Y. The function f is said to be many-one functions if there exist two or more than two different elements in X having the same image in Y.

Example:

Consider X = {1, 2, 3, 4, 5}

Y = {x, y, z} and f: X → Y such that

f = {(1, x), (2, x), (3, x), (4, y), (5, z)}



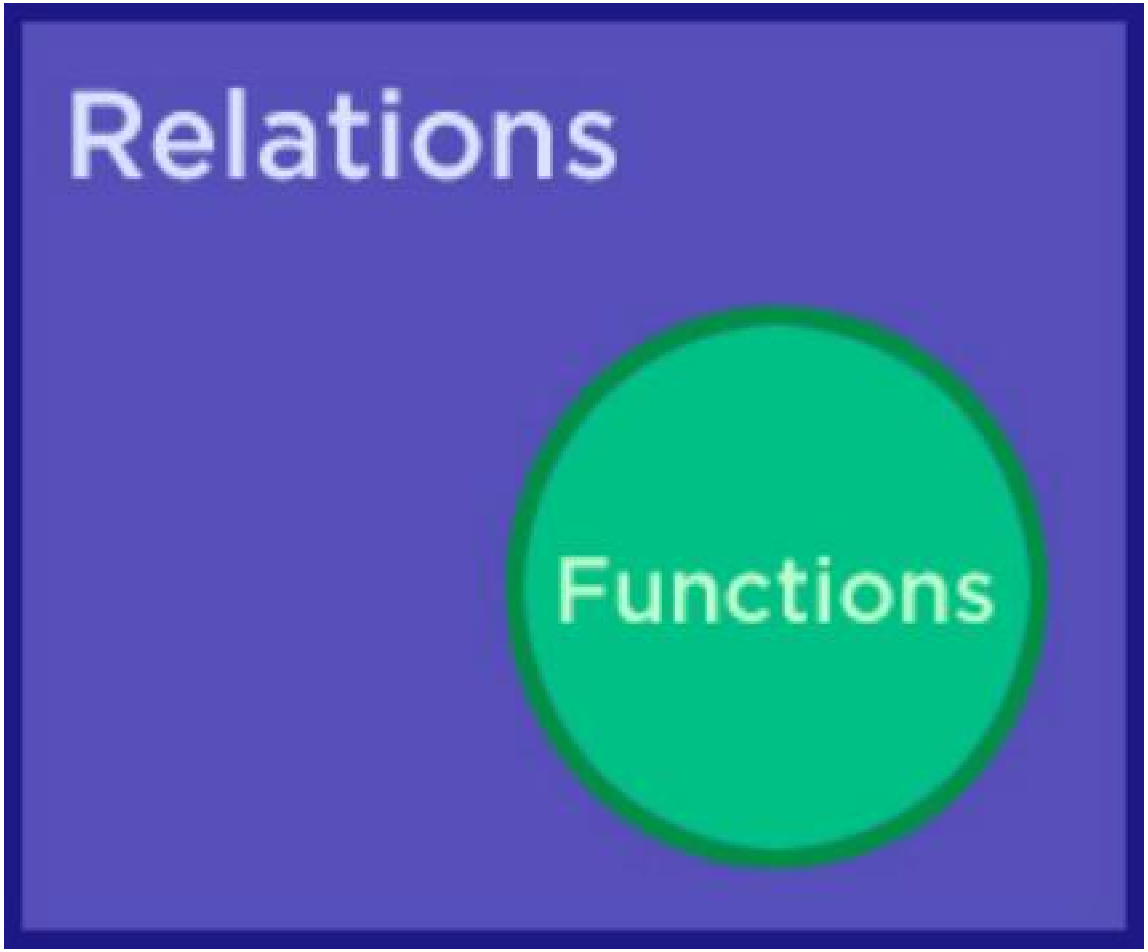
6. Even function:

- The mathematical definition of an even function is

  $f(-x) = f(x)$ for any value
- The simplest example of this is $f(x) = x2$; $f(3) = 9$, and $f(-3) = 9$.
- Basically, the opposite input yields the same output.

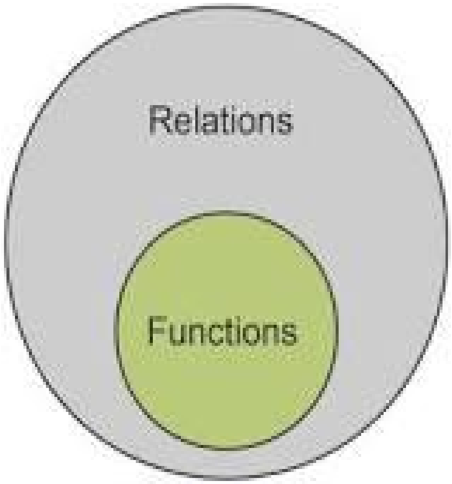
7. Odd function:

- The definition of an odd function is $f(-x) = -f(x)$ for any value.
- The opposite input gives the opposite output.
- For example, $f(x) = x3$ is an odd function because $f(3) = 27$ and $f(-3) = 27$.


Relations
Functions

INTRODUCTION - RELATION

- A **relation** is a set of inputs and outputs
- A **function** is a relation with one output for each input. (or)
- every X-value should be associated to only one y-value is called a Function.



- Domain → Collection of the first value in Ordered pair (X value)
- Range → Collection of second value in Ordered pair (Y value)

CARTESIAN PRODUCT OF THE SETS

- The set of all possible ordered pairs (a,b) with a ∈ A and b ∈ B is called cartesian product of A and B
- Denoted by → A x B

A x B ={ (a,b | a∈A, b∈B}

NOTES

1. $A \times B = \{ (a,b \mid a \in A, b \in B\}$
   Where, $a \in A$
   $b \in B$

2. $A \times B \neq B \times A$

3. $n(A) = P$, $n(B)=Q$
   $n(A \times B) = PQ$

4. $A = \varnothing$ or $B = \varnothing$
   $A \times B = \varnothing$

5. $A = \{1,2,3,4\ldots\}$ $B = \{1,2,3\}$
   $A \times B = \{(1,1), (1,2), (1,3) \ldots\ldots\}$

EXAMPLE:

1. Let A = {1, 2, 3} and B = {4, 5}. Find A x B and B x A and show that A x B ≠ B x A.

Solution:

AxB = {(1, 4) (1, 5) (2, 4) (2, 5) (3, 4) (3, 5)}

B x A = {(4, 1) (4, 2) (4, 3) (5, 1) (5, 2) (5, 3)}

From the above (1, 4) ∈ A x B

but

(1, 4) ∉ B x A.

So, A x B ≠ B x A.

REPRESENTATION OF A RELATION

- A relation is represented either by Roster method or by Set-builder method.

- Consider an example of two sets A = {9, 16, 25} and B = {5, 4, 3, -3, -4, -5}. The relation is that the elements of A are the square of the elements of B.

- In set-builder form, R = {(x, y): x is the square of y, $x \in A$ and $y \in B$}.

- In roster form, R = {(9, 3), (9, -3), (16, 4), (16, -4), (25, 5), (25, -5)}.

Terminologies

- **Domain:** The set of all first elements of the ordered pairs in a relation R from a set A to a set B.

- **Range:** The set of all second elements in a relation R from a set A to a set B.

- **Codomain:** The whole set B. Range ⊆ Codomain.

EXAMPLE:

1. Let A = {5, 6, 7, 8, 9, 10} and B = {7, 8, 9, 10, 11, 13}. Define a relation R from A to B by R = {(x, y): y = x + 2}. Write down the domain, codomain and range of R.

Solution:

i). x= 5, y=?	(ii) x=6, y=?
$y = x + 2$	$y = x + 2$
$= 5+2$	$= 6+2$
$y = 7$	$y = 8$

(iii) x=8

y= x+2

= 8+2

y = 10

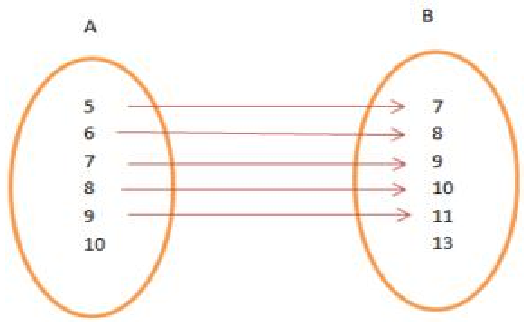
(iv) x = 9

y = x + 2

= 9 + 2

y = 11

Here, R = {(5, 7), (6, 8), (7, 9), (8, 10), (9, 11)}.



- Domain = {5, 6, 7, 8, 9}
- Range = {7, 8, 9, 10, 11}
- Co-domain = {7, 8, 9, 10, 11, 13}.

TYPES OF RELATION

1. Inverse Relation

Inverse relation is seen when a set has elements which are inverse pairs of another set.

For example

if set A = {(a, b), (c, d)}, then

inverse relation will be R-1 = {(b, a), (d, c)}.

So, for an inverse relation,

R-1 = {(b, a): (a, b) ∈ R}

2. Reflexive Relation

In a reflexive relation, every element maps to itself.

For example,

consider a set A = {1, 2,}.

Now an example of reflexive relation will be

R = {(1, 1), (2, 2), (1, 2), (2, 1)}.

The reflexive relation is given by-

$(a, a) \in R$

3. Symmetric Relation

In a symmetric relation, if a=b is true then b=a is also true.

In other words, a relation R is symmetric only if (b, a) ∈ R is true when (a,b) ∈ R.

An example of symmetric relation will be

R = {(1, 2), (2, 1)} for a set A = {1, 2}.

So, for a symmetric relation,

$aRb \Rightarrow bRa, \forall\ a, b \in A$

4. Transitive Relation

For transitive relation, if $(x, y) \in R$, $(y, z) \in R$, then $(x, z) \in R$.

For a transitive relation,

aRb and $bRc \Rightarrow aRc \; \forall \; a, b, c \in A$

Equivalence Relation:

- A relation R on a set A is said to be an equivalence relation if and only if the relation R is reflexive, symmetric and transitive.

Reflexive: A relation is said to be reflexive, if (a, a) ∈ R, for every a ∈ A.

Symmetric: A relation is said to be symmetric, if (a, b) ∈ R, then (b, a) ∈ R.

Transitive: A relation is said to be transitive if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.