Bharathidasan University

Centre for Differently Abled Persons

Tiruchirappalli - 620024.

- Programme Name : Bachelor of Computer Applications
- Course Code : Discrete Mathematics
- Course Title : 20UCA1AC1
- Unit : Unit III
- Compiled by : Dr. M. Prabavathy
  Associate Professor
  Ms. G. Maya Prakash
  Guest Faculty

MATHEMATICAL LOGIC

TYPES OF PROPOSITION

(i) TAUTOLOGIES

Prove $[(A \to B) \wedge A] \to B$ is a tautology.

The truth table is as follows:

A	B	$A \to B$	$(A \to B) \wedge A$	$[(A \to B) \wedge A] \to B$
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True

Example 1:

Show that (P→Q) V (Q→P) is a tautology.

construct the truth table for (P→Q) V (Q→P) and show that the formula is always true.

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \vee (Q \to P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(ii). CONTRADICTIONS

A Contradiction is a formula which is always false for every value of its propositional variables.

Prove $(A \lor B) \land [(\neg A) \land (\neg B)]$ is a contradiction

A	B	A ∨ B	¬ A	¬ B	(¬ A) ∧ (¬ B)	(A ∨ B) ∧ [(¬ A) ∧ (¬ B)]
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

$A \lor B) \land [(\neg A) \land (\neg B)]$ is "False", it is a contradiction.

Example 1:

Show that the statement $p \wedge \sim p$ is a contradiction.

Solution:

p	**~ p**	**p ∧~ p**
T	F	F
F	T	F

LOGICAL EQUIVALENCE

- Two propositions **P** and **Q** are said to be logically equivalent if is a **Tautology**.
- The notation ≡ is used to denote that **P** and **Q** are logically equivalent.

$$P \equiv Q$$

Prove: Show that $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalent.

P	Q	$P \rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

CONVERSE

- The converse of the conditional statement is interchanging the hypothesis (p) and the conclusion (q).
- If the statement is "If p, then q", the converse will be "If q, then p".
- The converse of $p \rightarrow q$ is $q \rightarrow p$.

p	q	p→q	q→p	p
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	F

MATHEMATICAL LOGIC

Logical statement of proposition

- \- A proposition is a statement that can be either true or false; it must be one or the other, and it cannot be both

    true (denoted either T or 1) or

    false (denoted either F or 0).

- Propositional variables are represented by small alphabets such as **p, q, r, s**
- The area of logic which deals with propositions is called **propositional calculus or propositional logic.**
- Propositions constructed using one or more propositions are called **compound propositions.**
- The propositions are combined together using **Logical Connectives or Logical Operators.**

Basic logical operator

1. Negation/ NOT (¬)

- Negation – If p is a proposition, then the negation of p is denoted by ¬ p
- The truth value of ¬p is the opposite of the truth value of p.
- The truth table of ¬p is-

p	**¬p**
T	F
F	T

2. Implication or Conditional Proposition

- For any two propositions p and q, the statement "if p then q" is called an implication and it is denoted by $p \rightarrow q$.

- The implication is $p \rightarrow q$ is also called a conditional statement.

- The implication is false when p is true and q is false otherwise it is true.

The truth table of $p \rightarrow q$ is-

p	**q**	**$p \rightarrow q$**
T	T	T
T	F	F
F	T	T
F	F	T

3. Conjunction

- For any two propositions p and q, their conjunction is denoted by p^q, which means "p and q"

- The conjuction p^ q is True when both p and q are True, otherwise False.

The truth table of p^ q is-

p	**q**	**p^q**
T	T	T
T	F	F
F	T	F
F	F	F

4. Disjunction

- For any two propositions p and q, their disjunction is denoted by pVq, which means “p or q”.

- The disjuction p V q is True when either p or q is True, otherwise False.

The truth table of p V q is-

p	**q**	**p V q**
T	T	T
T	F	T
F	T	T
F	F	F

5. If and Only If (⟷):

- p⟷q is bi-conditional logical connective which is true when p and q are same, i.e., both are false or both are true.

p	**q**	**p⟷q**
T	T	T
T	F	F
F	T	F
F	F	T

INVERSE

- An inverse of the conditional statement is the negation of both the hypothesis(p) and the conclusion(q).
- If the statement is "If p, then q", the inverse will be "If not p, then not q".
- Thus, the inverse of $p \rightarrow q$ is $\neg p \rightarrow$

p	q	¬ p	¬ q	¬ p → ¬ q.
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

CONTRA-POSITIVE

- The contra-positive of the conditional is interchanging the hypothesis and the conclusion of the inverse statement.
- If the statement is "If p, then q", the contra-positive will be "If not q, then not p".
- The contra-positive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

p	**q**	**¬ q**	**¬ p.**	**¬ q → ¬ p.**
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

THE NEGATION OF CONDITIONAL STATEMENT

- The negation of a conditional statement is only true when the original if-then statement is false.

P	Q	$P \rightarrow Q$	$\neg (P \rightarrow Q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

BICONDITIONAL STATEMENT

- If p and q are two statements then "p if and only if q" is a compound statement.
- denoted as **$p \leftrightarrow q$** and referred as a biconditional statement or an equivalence.
- The equivalence $p \leftrightarrow q$ is true only when both p and q are true or when both p and q are false.

p	**q**	**$p \leftrightarrow q$**
T	T	T
T	F	F
F	T	F
F	F	T

ARGUMENTS

- An argument form, or argument for short, is a sequence of statements.

- All statements but the last one is called premises or hypotheses.

- The final statement is called the conclusion, and is often preceded by a symbol "   ".

- An argument is valid if the conclusion is true whenever all the premises are true.

- The validity of an argument can be tested through the use of the truth table by checking if the critical rows.

Example 1:

Show that (p   q, p   r, q   r,   r) is a valid argument.

Solution:

p	q	r	$p \vee q$	$p \to r$	$q \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

critical rows

not a critical row

all critical rows (in this case, those with the shaded positions all containing a *T*) correspond to (the circled) T(true) for r.

Hence the argument is valid.

Example 2:

Show that the argument ($p \rightarrow q$, ∴ $\sim p \rightarrow \sim q$) is invalid.

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \rightarrow \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

this critical row fails

On the 3rd row, a critical row, the premise p q is true while the conclusion p   q is false.

Hence the argument (**$p \rightarrow q$, ∴ $\sim p \rightarrow \sim q$**) is invalid