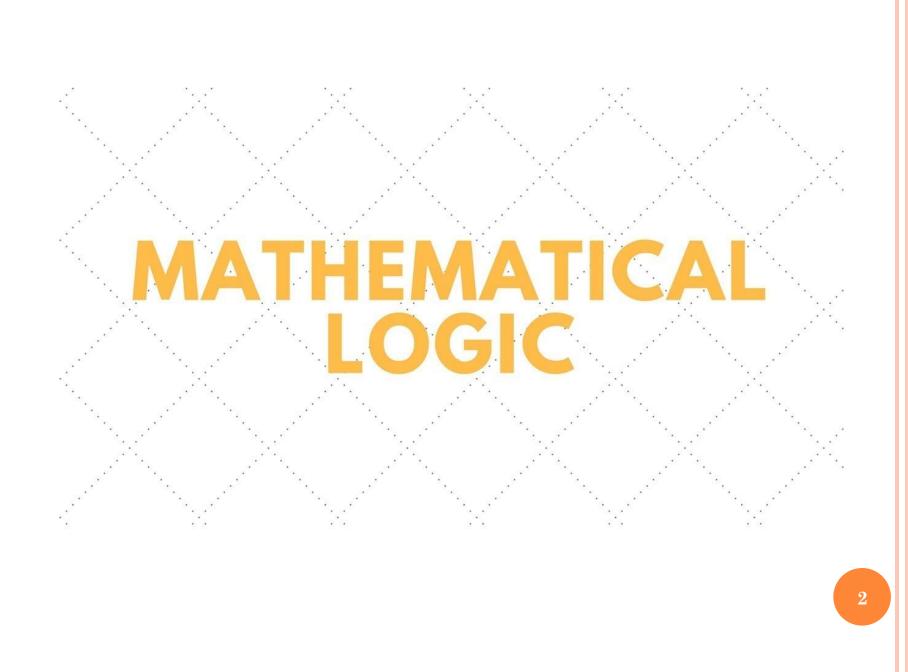


BHARATHIDASAN UNIVERSITY

CENTRE FOR DIFFERENTLY ABLED PERSONS TIRUCHIRAPPALLI - 620024.

- Programme Name : Bachelor of Computer Applications
- Course Code : Discrete Mathematics
- Course Title : 20UCA1AC1
- Unit : Unit III
- Compiled by

: Dr. M. Prabavathy Associate Professor Ms. G. Maya Prakash Guest Faculty



TYPES OF PROPOSITION

(i) TAUTOLOGIES

Prove $[(A \rightarrow B) \land A] \rightarrow B$ is a tautology.

The truth table is as follows:

A	В	$\mathbf{A} \rightarrow \mathbf{B}$	$(\mathbf{A} \rightarrow \mathbf{B}) \wedge \mathbf{A}$	$\begin{bmatrix} (A \to B) \land A \end{bmatrix} \to B$
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True

Example 1:

Show that $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology.

construct the truth table for $(P \rightarrow Q) \vee (Q \rightarrow P)$ and show that the formula is always true.

Р	Q	$P \to Q$	$Q \to P$	$(P \to Q) \lor (Q \to P)$
Т	Т	Т	Т	Т
Т	F	F	Т	Т
F	Т	Т	F	Т
F	F	Т	Т	Т

(ii). CONTRADICTIONS

A Contradiction is a formula which is always false for every value of its propositional variables.

Prove $(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$ is a contradiction

Α	В	A V B	ΓA	¬ B	(¬A)∧(¬ B)	(A ∨ B) ∧ [(¬ A) ∧ (¬ B)]
Т	Т	Т	F	F	F	F
Т	F	Т	F	Т	${f F}$	F
F	Т	Т	Т	F	${f F}$	F
F	F	F	Т	Т	Т	F

AVB) $\Lambda[(\neg A)\Lambda(\neg B)]$ is "False", it is a contradiction.

Example 1: Show that the statement $p \wedge p$ is a contradiction.

Solution:

р	~ p	р∧~р
Т	F	F
F	Т	F

LOGICAL EQUIVALENCE

- Two propositions **P** and **Q** are said to be logically equivalent if is a **Tautology**.
- The notation \equiv is used to denote that **P** and **Q** are logically equivalent.

 $\mathbf{P} \equiv \mathbf{Q}$

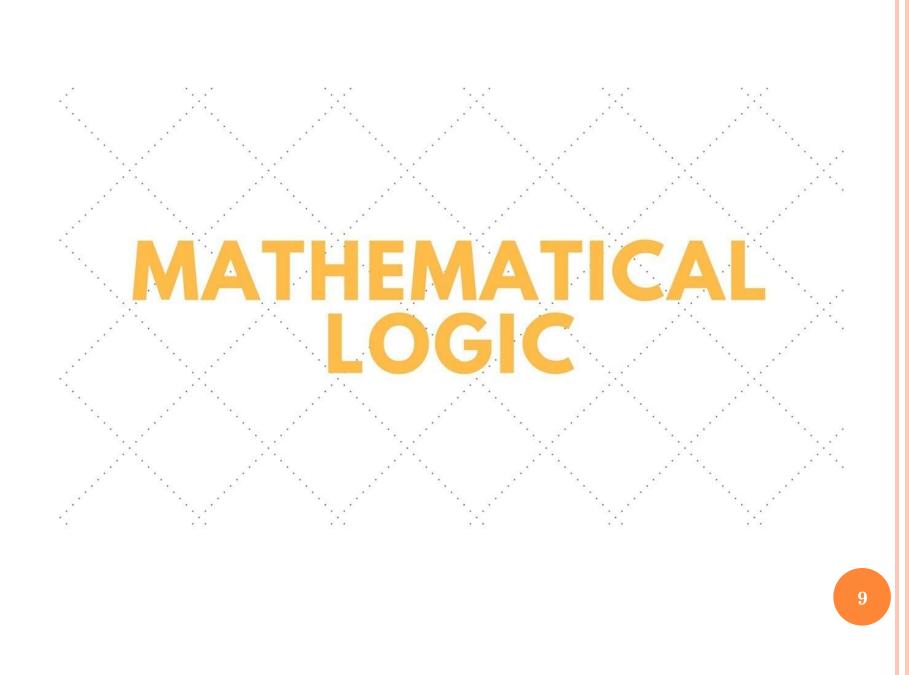
Prove: Show that $P \rightarrow Q$ and $\neg P \lor Q$ are logically equivalent.

Р	Q	$P \to Q$	$\neg P$	$\neg P \lor Q$
Т	Т	Т	F	Т
Т	F	F	F	F
F	Т	Т	Т	Т
F	F	Т	Т	Т

CONVERSE

- The converse of the conditional statement is interchanging the hypothesis (p) and the conclusion (q).
- If the statement is "If p, then q", the converse will be "If q, then p".
- The converse of $p \rightarrow q$ is $q \rightarrow p$.

р	q	թ→զ	q→p	р
Т	Т	Т	Т	Т
Т	F	F	Т	Т
F	Т	Т	F	F
F	F	Т	Т	F



LOGICAL STATEMENT OF PROPOSITION

• A proposition is a statement that can be either true or false; it must be one or the other, and it cannot be both

true (denoted either T or 1) or

false (denoted either F or 0).

- Propositional variables are represented by small alphabets such as **p**, **q**, **r**, **s**
- The area of logic which deals with propositions is called **propositional calculus or propositional logic.**
- Propositions constructed using one or more propositions are called **compound propositions**.
- The propositions are combined together using Logical Connectives or Logical Operators.

BASIC LOGICAL OPERATOR

1. Negation/ NOT (¬)

- Negation If p is a proposition, then the negation of p is denoted by ¬ p
- The truth value of ¬p is the opposite of the truth value of p.
- The truth table of ¬p is-

р	¬р
Т	F
F	Т

2. Implication or Conditional Proposition

- For any two propositions p and q, the statement "if p then q" is called an implication and it is denoted by p→ q.
- The implication is p→ q is also called a conditional statement.
- The implication is false when p is true and q is false otherwise it is true.

The truth table of $p \rightarrow q$ is-

р	q	p→q
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

12

3. Conjunction

- For any two propositions p and q, their conjunction is denoted by p[^]q, which means "p and q"
- The conjuction p[^] q is True when both p and q are True, otherwise False.

The truth table of p[^] q is-

р	q	p^q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

4. Disjunction

- For any two propositions p and q, their disjunction is denoted by pVq, which means "p or q".
- The disjuction p V q is True when either p or q is True, otherwise False.

The truth table of p V q is-

р	q	p V q
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

5. If and Only If (\leftrightarrow) :

• p↔q is bi-conditional logical connective which is true when p and q are same, i.e., both are false or both are true.

р	q	p↔q
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

INVERSE

- An inverse of the conditional statement is the negation of both the hypothesis(p) and the conclusion(q).
- If the statement is "If p, then q", the inverse will be "If not p, then not q".

р	q	¬ p	¬ q	$\neg p \rightarrow \neg q.$
Т	Т	F	F	Т
Т	F	F	Т	Т
F	Т	Т	F	F
F	F	Т	Т	Т

• Thus, the inverse of $p \rightarrow q$ is $\neg p \rightarrow q$

CONTRA-POSITIVE

- The contra-positive of the conditional is interchanging the hypothesis and the conclusion of the inverse statement.
- If the statement is "If p, then q", the contra-positive will be "If not q, then not p".
- The contra-positive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

р	q	¬ q	¬ p.	$\neg q \rightarrow \neg p.$
Т	Т	F	F	Т
Т	F	Т	F	F
F	Т	F	Т	Т
F	F	Т	Т	Т

THE NEGATION OF CONDITIONAL STATEMENT

• The negation of a conditional statement is only true when the original if-then statement is false.

Р	Q	P→Q	$\neg (P \rightarrow Q)$
Т	Т	Т	F
Т	F	F	Т
F	Т	Т	F
F	F	Т	F

BICONDITIONAL STATEMENT

- If p and q are two statements then "p if and only if q" is a compound statement.
- o denoted as p ↔ q and referred as a biconditional statement or an equivalence.
- The equivalence p ↔ q is true only when both p and q are true or when both p and q are false.

р	q	$\mathbf{p} \leftrightarrow \mathbf{q}$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

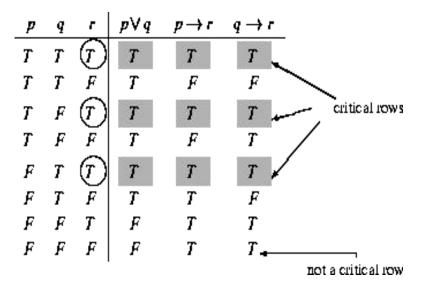
ARGUMENTS

- An argument form, or argument for short, is a sequence of statements.
- All statements but the last one is called premises or hypotheses.
- The final statement is called the conclusion, and is often preceded by a symbol " ".
- An argument is valid if the conclusion is true whenever all the premises are true.
- The validity of an argument can be tested through the use of the truth table by checking if the critical rows.

Example 1:

Show that (p q, p r, q r, r) is a valid argument.

Solution:

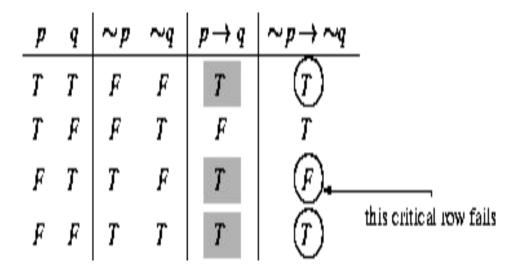


all critical rows (in this case, those with the shaded positions all containing a T) correspond to (the circled) T(true) for r.

Hence the argument is valid.

Example 2:

Show that the argument $(p \rightarrow q, \neg p \rightarrow \neg q)$ is invalid.



On the 3rd row, a critical row, the premise p q is true while the conclusion p q is false.

Hence the argument ($p \rightarrow q$, $\neg p \rightarrow \neg q$) is invalid