Bharathidasan University

Centre for Differently Abled Persons

Tiruchirappalli - 620024.

- Programme Name : Bachelor of Computer Applications
- Course Code : Discrete Mathematics
- Course Title : 20UCA1AC1
- Unit : Unit IV
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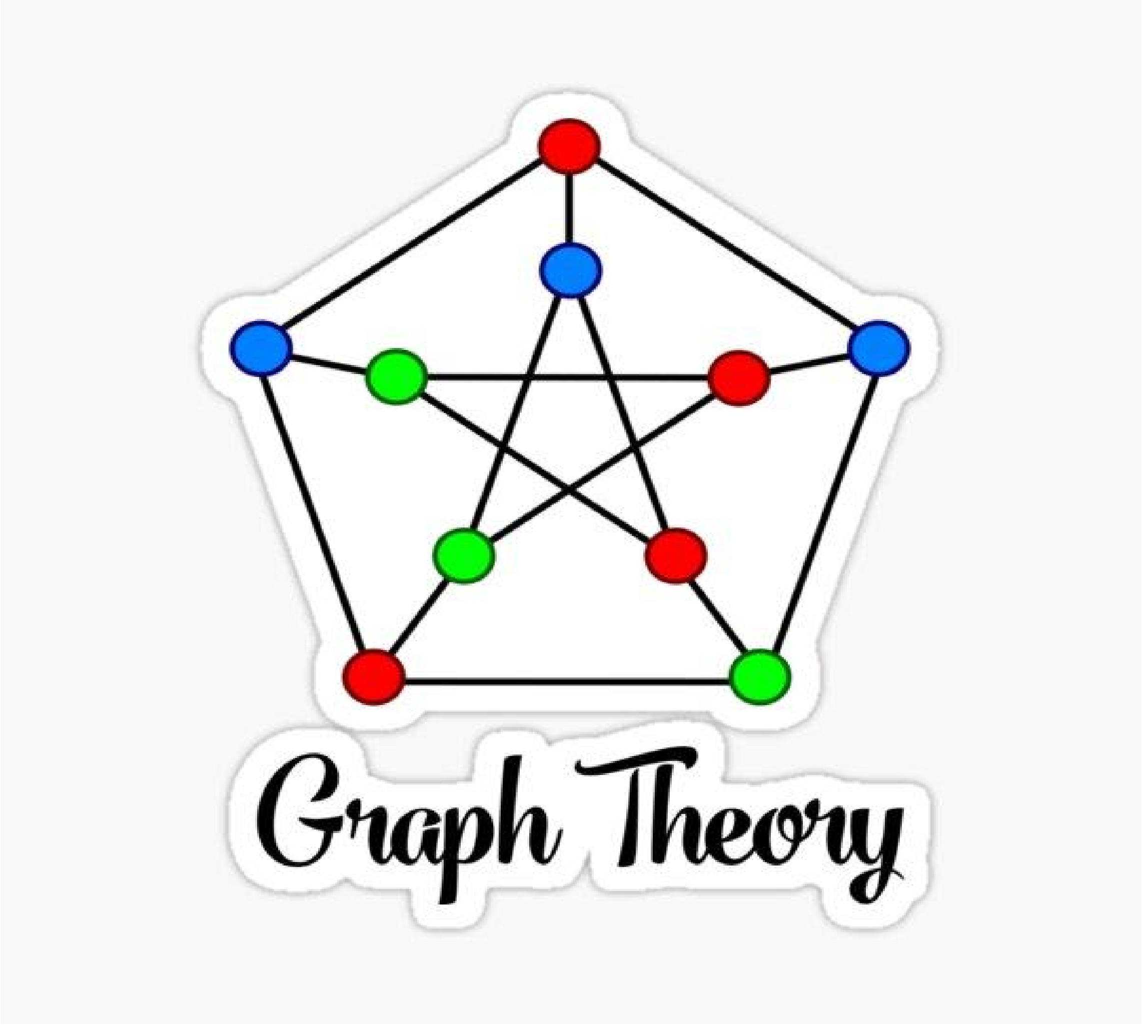
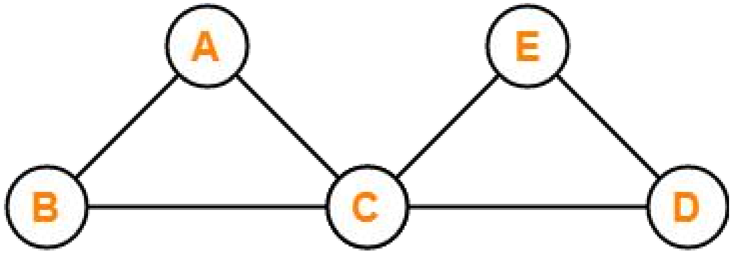

Graph Theory

EULER GRAPH

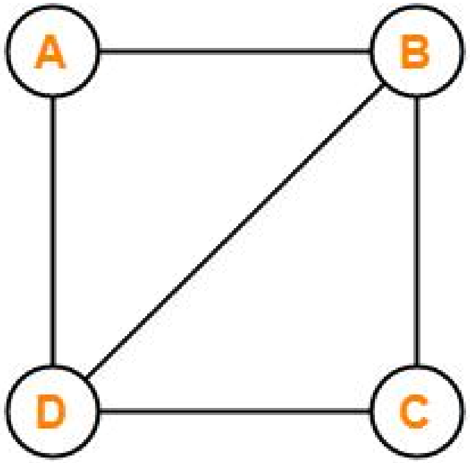
- A Euler Graph is a connected graph whose all vertices are of even degree.



- This graph is a connected graph and all its vertices are of even degree.
- Therefore, it is a Euler graph.
- Graph contains a Euler circuit BACEDCB, so it is an Euler graph.

EULER PATH

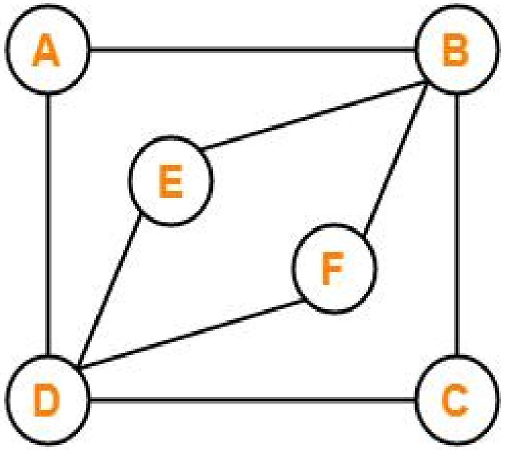
- A Euler path is a path that uses every edge of a graph exactly once.
- A Euler path starts and ends at different vertices.



Euler Path = BCDBAD

EULER CIRCUIT

- A Euler circuit is a circuit that uses every edge of a graph exactly once.
- A Euler circuit always starts and ends at the same vertex.



Euler Circuit = ABCDFBEDA

OPERATION ON GRAPH

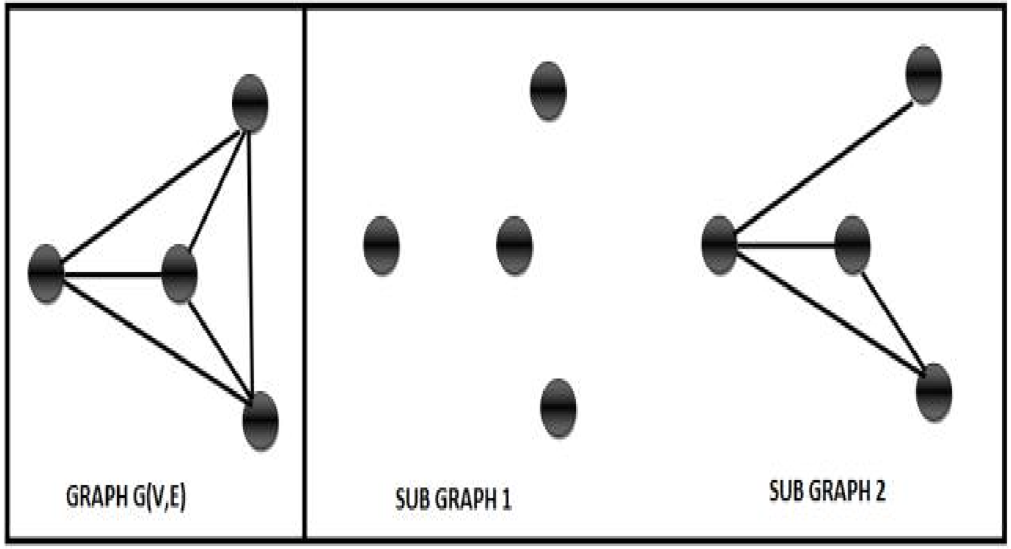
1. Sub graph

A sub graph of a graph G(V,E) can be obtained by the following means:

Removing one or more vertices from the vertex set.

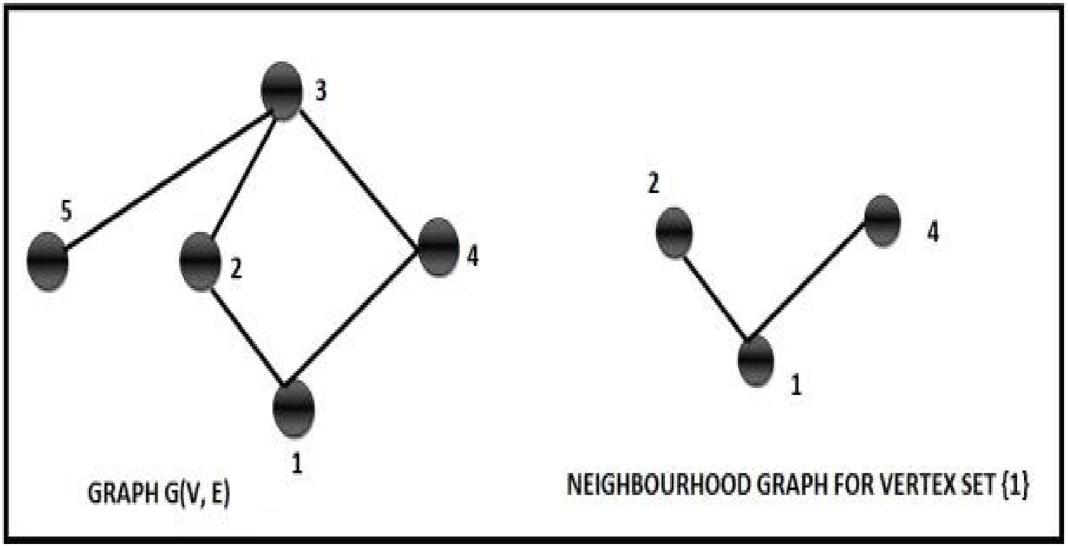
Removing one or more edges from the edge family.

Removing either vertices or edges from the graph.



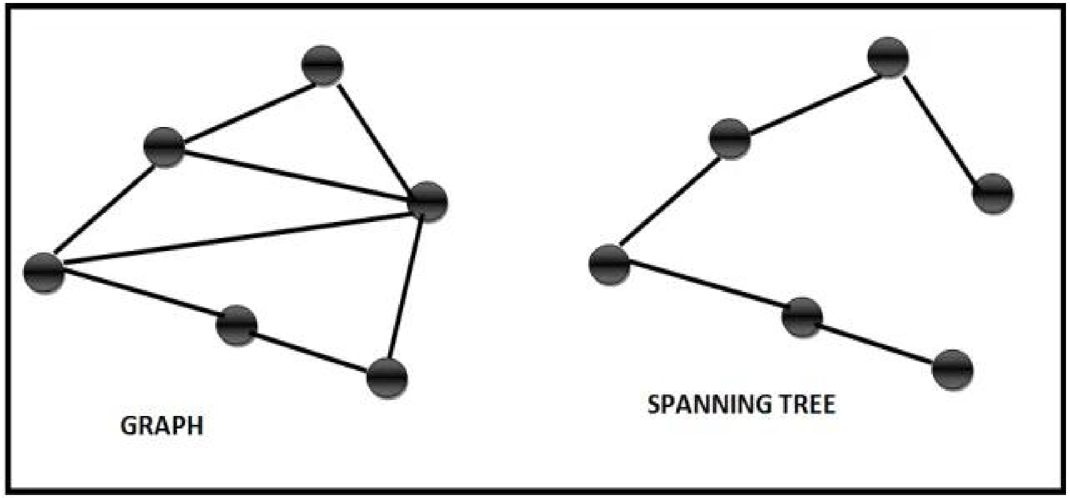
2. Neighbourhood graph

- The neighbourhood graph of a graph G(V,E) mention it with respect to a given vertex set.
- For e.g. if V = {1,2,3,4,5} then we can find out the Neighbourhood graph of G(V, E) for vertex set {1}.



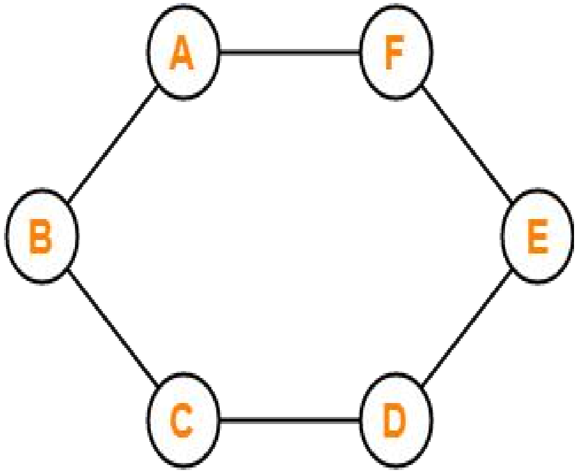
3. SPANNING TREE

- A spanning tree of a connected graph G(V,E) is a sub graph that is also a tree and connects all vertices in V.
- For a disconnected graph the spanning tree would be the spanning tree of each component.



HAMILTONIAN GRAPH

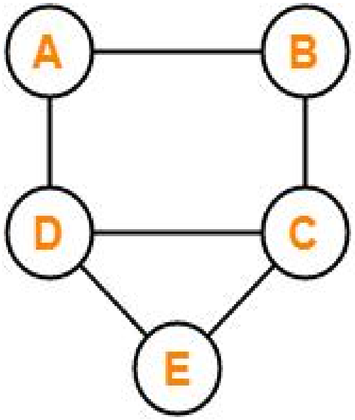
- Connected graph that visits every vertex of the graph exactly once without repeating the edges, then such a graph is called as a Hamiltonian graph.



- This graph contains a closed walk ABCDEFA.
- It visits every vertex of the graph exactly once except starting vertex.
- The edges are not repeated during the walk.
- Therefore, it is a Hamiltonian graph.

HAMILTON IAN PATH

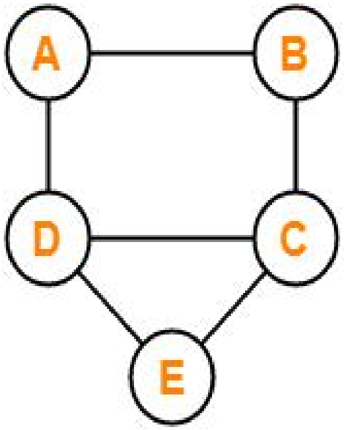
- If there exists a walk in the connected graph that visits every vertex of the graph exactly once without repeating the edges, then such a walk is called as a **Hamiltonian path.**



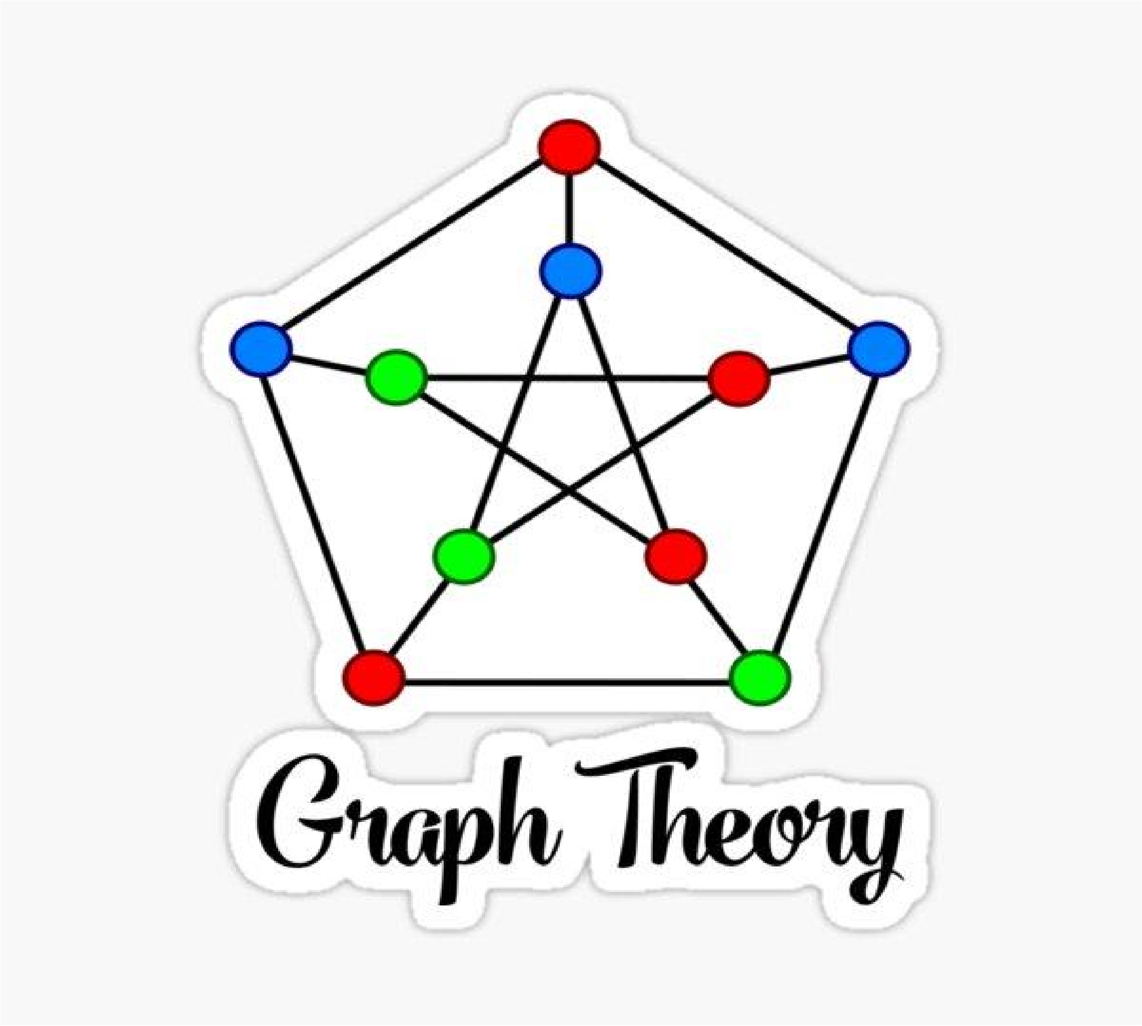
Hamiltonian Path = ABCDE

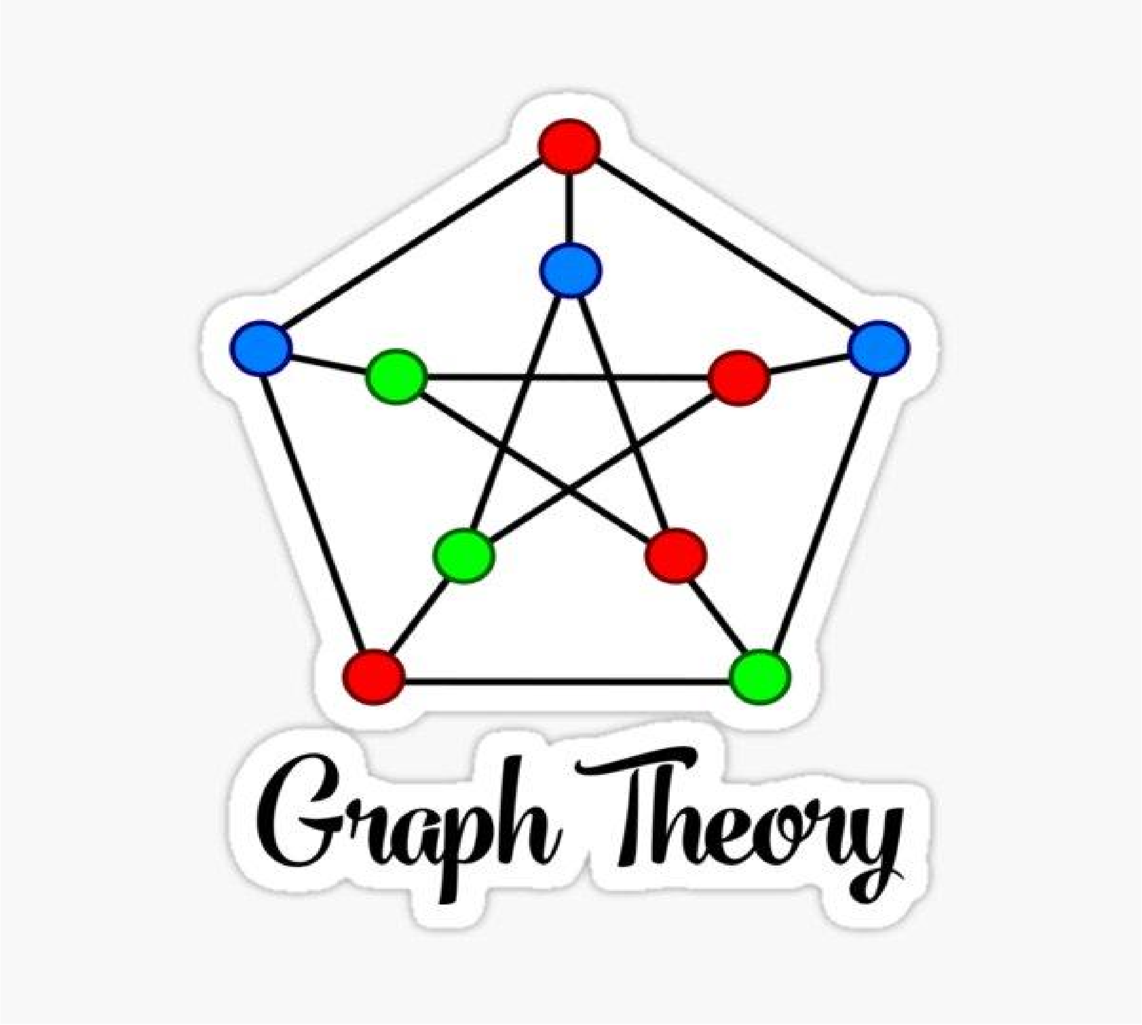
HAMILTONIAN CIRCUIT

- A Hamiltonian path which starts and ends at the same vertex is called as a Hamiltonian circuit.



Hamiltonian Circuit = ABCEDA

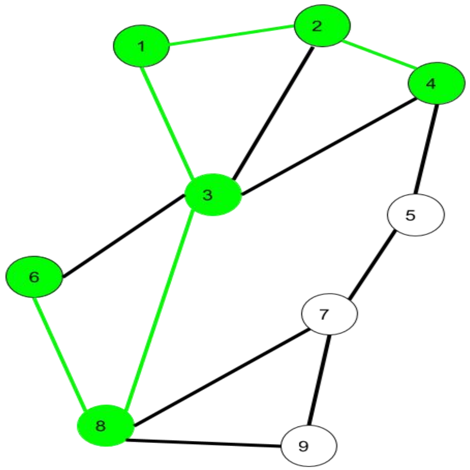

Graph Theory


Graph Theory

PATH

- A path is a sequence of vertices with the property that each vertex in the sequence is adjacent to the vertex next to it.

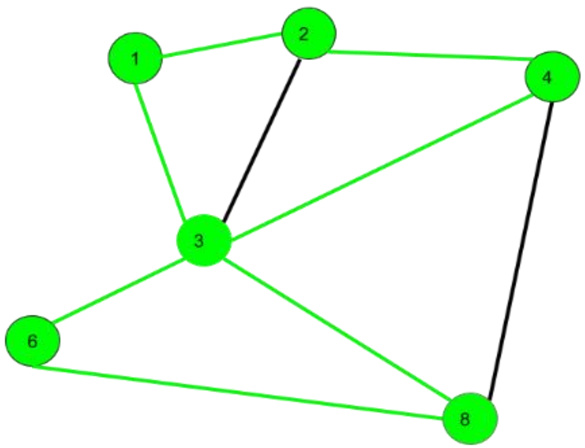
- A path that does not repeat vertices is called a simple path.



- Here 6->8->3->1->2->4 is a Path

CIRCUIT

- A circuit is path that begins and ends at the same vertex.
- A circuit that doesn't repeat vertices is called a cycle.



Here 1->2->4->3->6->8->3->1 is a circuit.

WALK

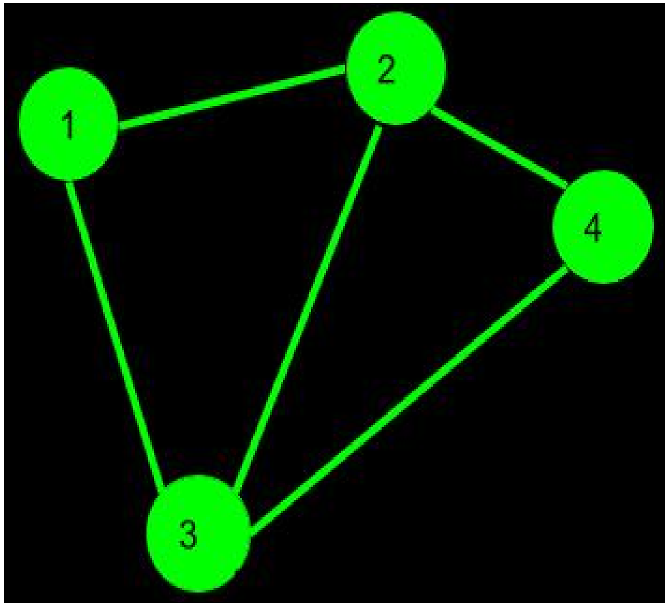
- A walk is a sequence of vertices and edges of a graph i.e. if we traverse a graph then we get a walk.

Vertex can be repeated

Edges can be repeated

(i) Open walk-A walk is said to be an open walk if the starting and ending vertices are different

(ii) Closed walk-A walk is said to be a closed walk if the starting and ending vertices are same.



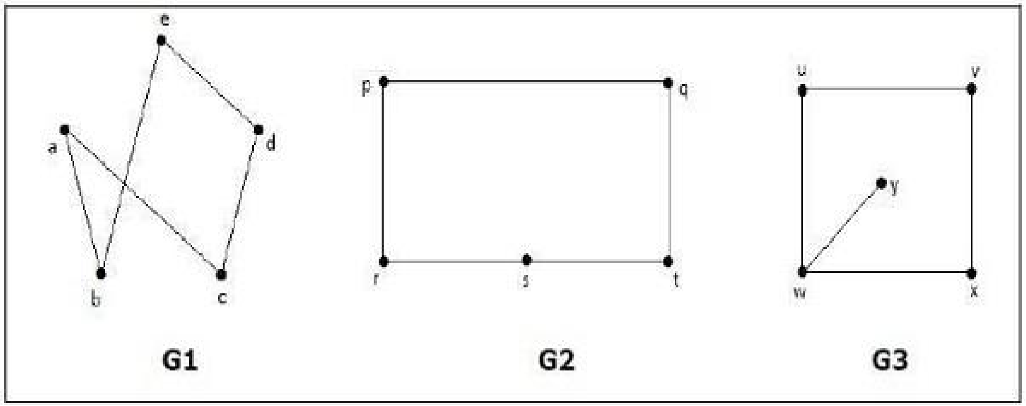
Here, 1->2->3->4->2->1->3 is a walk.

1->2->3->4->5->3-> is an open walk.

1->2->3->4->5->3->1-> is a closed walk.

ISOMORPHISM

- Two graphs G1 and G2 are said to be isomorphic if –
- Their number of components (vertices and edges) are same.
- Their edge connectivity is retained.



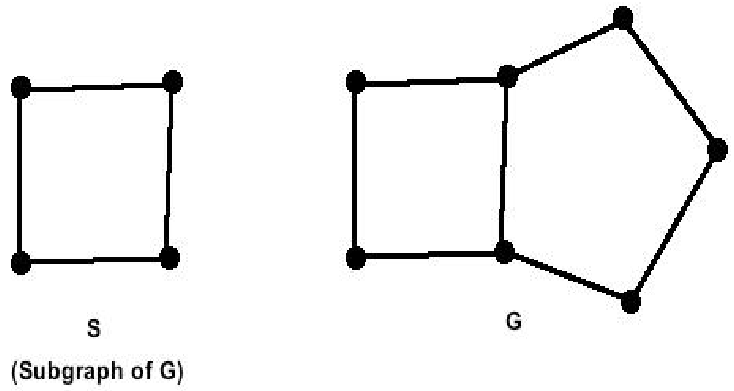
- In the graph G3, vertex ‘w’ has only degree 3,
- whereas all the other graph vertices have degree 2.
- Hence G3 not isomorphic to G1 or G2.

SUB GRAPH

A graph whose vertices and edges are subsets of another graph.

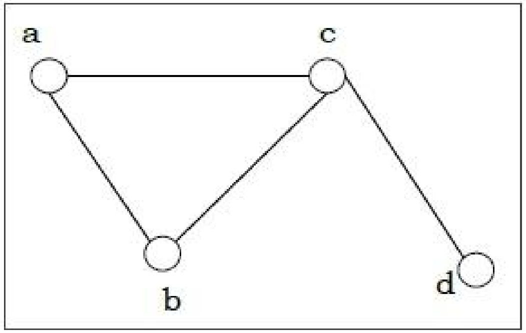
A graph G'=(V', E') is a subgraph of another graph G=(V, E)
iff $V' \subseteq V$, and

$E' \subseteq E \wedge ((v1, v2) \in E' \rightarrow v1, v2 \in V')$.



CONNECTED GRAPH

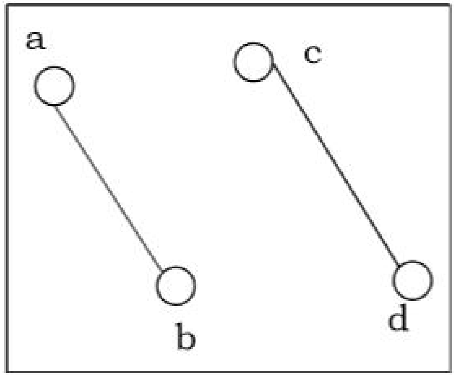
A graph is connected if any two vertices of the graph are connected by a path.



Vertex 1	Vertex 2	PATH
a	b	a b
a	c	a b c, a c
a	d	a b c d, a c d
b	c	b a c , b c
c	d	c d

DISCONNECTED GRAPH

- A graph is disconnected if at least two vertices of the graph are not connected by a path.



Vertex 1	Vertex 2	PATH
a	b	a b
a	c	Not Available
a	d	Not Available
b	c	Not Available
c	d	c d

GRAPH THEORY

- A graph is a data structure that is defined by two components:

i) A **node or a vertex V**

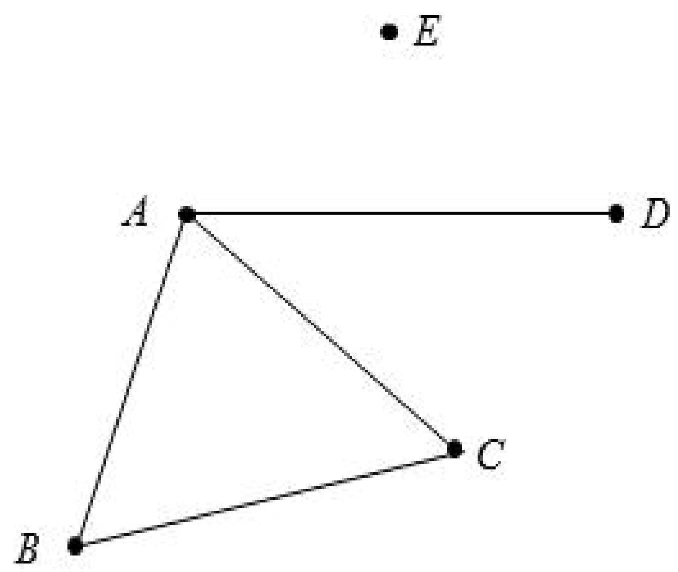
ii). An **edge** E or ordered pair is a connection between two nodes u,v that is identified by unique pair(u,v).

- The pair (u,v) is ordered because (u,v) is not same as (v,u) in case of directed graph.

A graph is an ordered pair **G = (V, E)** where,

G specifies the **graph.**

- **V** is the vertex-set whose elements are called the **vertices, or nodes** of the graph.
- **E** is the edge-set whose elements are called the **edges**, or connections between vertices of the graph.



V = {A, B, C, D, E}

E = {AB, BC, CA, AD}

DEFINITION:

Graph theory is the study of graphs, which are mathematical structures used to model pair wise relations between objects.

A graph in this context is made up of **vertices** (also called nodes or points) which are connected by **edges** (also called links or lines).

APPLICATION OF GRAPH

1. In mathematics, operational research is the important field.

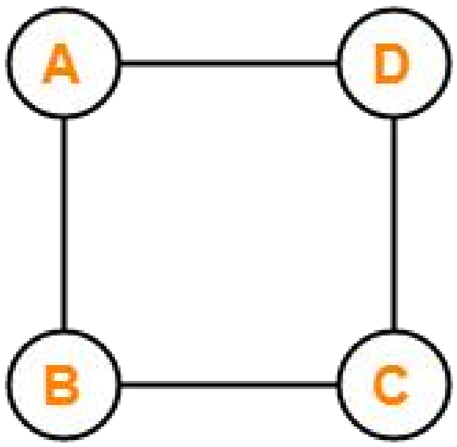
   Minimum cost path.

   A scheduling problem.

2. In computer science graph theory is used for the study of algorithms.
3. In physics and chemistry, graph theory is used to study molecules.
4. In computer network, the relationships among interconnected computers within the network
5. Graph theory is also used in network security

FINITE GRAPH

- A graph consisting of finite number of vertices and edges is called as a finite graph.

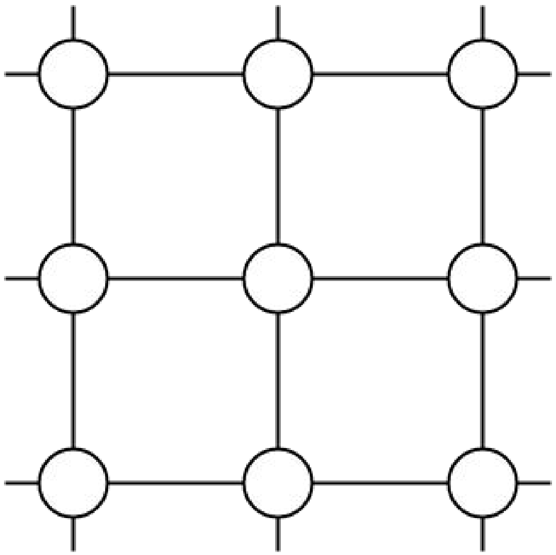


Example of Finite Graph

- Here,
- This graph consists of finite number of vertices and edges.
- Therefore, it is a finite graph.

INFINITE GRAPH:

- A graph consisting of infinite number of vertices and edges is called as an infinite graph.



Example of Infinite Graph

- Here,
- This graph consists of infinite number of vertices and edges.
- Therefore, it is an infinite graph.

INCIDENCE

- Two edges of a graph are called **adjacent** (sometimes coincident) if they share a common vertex.
- Similarly, two vertices are called adjacent if they share a common edge.
- An edge and a vertex on that edge are called **incident.**

DEGREE OF VERTEX

- It is the number of vertices adjacent to a vertex V.

  Notation − deg(V).
- In a simple graph with n number of vertices, the degree of any vertices is −

  $$\mathbf{deg(v) = n - 1 \ \forall \ v \in G}$$
- A vertex can form an edge with all other vertices except by itself.

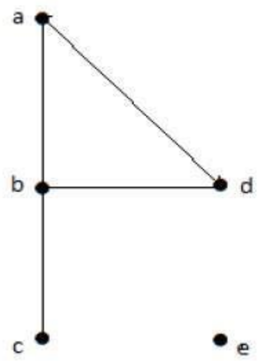
Degree of vertex can be considered under two cases of graphs –

Undirected Graph

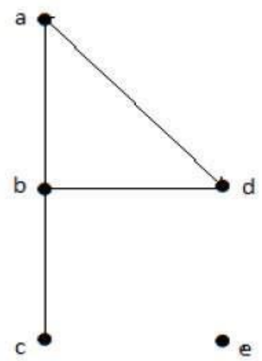
Directed Graph

(i). Undirected Graph

- An undirected graph has no directed edges.



Example:



In the above Undirected Graph,

deg(a) = 2, as there are 2 edges meeting at vertex 'a'.

deg(b) = 3, as there are 3 edges meeting at vertex 'b'.

deg(c) = 1, as there is 1 edge formed at vertex 'c'

So 'c' is a pendent vertex.

deg(d) = 2, as there are 2 edges meeting at vertex 'd'.

deg(e) = 0, as there are 0 edges formed at vertex 'e'.

So 'e' is an isolated vertex.

(ii). Directed Graph

In a directed graph, each vertex has an **indegree** and an **outdegree.**

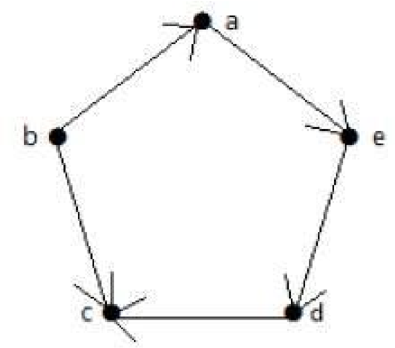
(a) **Indegree of vertex V** is the number of edges which are coming into the vertex V.

Notation − **deg−(V).**

(b) **Outdegree of vertex V** is the number of edges which are going out from the vertex V.

Notation − **deg+(V).**

EXAMPLE:



Vertex	Indegree	Outdegree
a	1	1
b	0	2
c	2	0
d	1	1
e	1	1

PENDENT VERTEX

- A vertex with degree one is called a pendent vertex.

Example

- Here, in this example, vertex 'a' and vertex 'b' have a connected edge 'ab'.

- vertex 'a', there is only one edge towards vertex 'b' and

- vertex 'b', there is only one edge towards vertex 'a'.

- Finally, vertex 'a' and vertex 'b' has degree as one which are also called as the pendent vertex.

ISOLATED VERTEX

- A vertex with degree zero is called an isolated vertex.

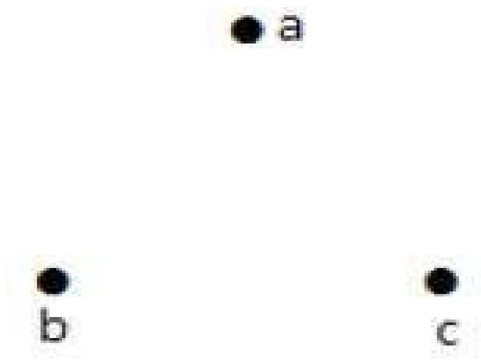
- Here, the vertex 'a' and vertex 'b' has a no connectivity between each other and also to any other vertices.

- So, the degree of both the vertices 'a' and 'b' are zero.

- These are also called as isolated vertices.

NULL GRAPH

- A graph having no edges is called a Null Graph.



- There are three vertices named ‘a’, ‘b’, and ‘c’, but there are no edges among them.

- Hence it is a Null Graph.