Bharathidasan University

Tiruchirappalli – 620 024, Tamil Nadu, India

Programme: M. Sc., Physics

Course Title : Electromagnetic Theory

Course Code : 22PH301

Unit I

Perspectives of Electrostatics

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Uniqueness Theorem

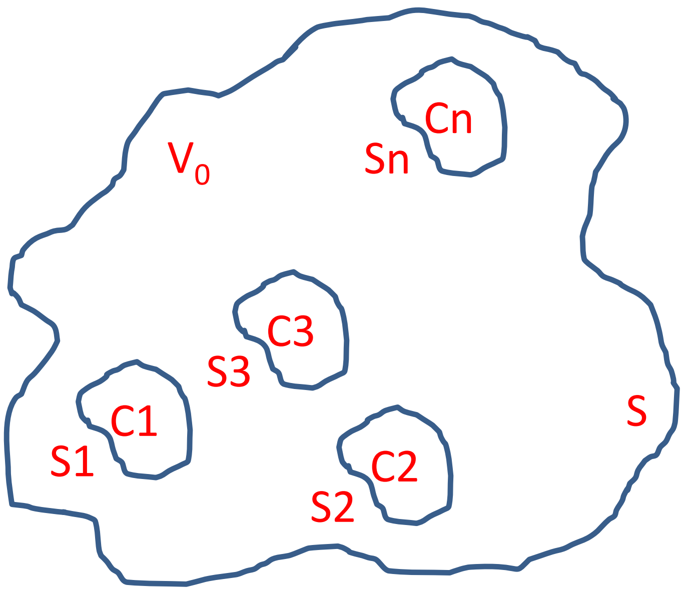
- Laplace equation does not itself determine the potential and hence a suitable boundary condition is required
- Uniqueness theorem states that "Laplace's equation satisfying given boundary conditions have one and only one (unique) solution"
- Consider a closed volume V0 exterior to the surfaces S1, S2…Sn of the various conductors C1, C2…Cn and bounded on the outside by a surface S.

Using Gauss divergence theorem

$$\int_{S1+S2+S3+\cdots+Sn} \emptyset \nabla \emptyset . dS = \int_{Vo} \nabla . (\emptyset \nabla \emptyset) . dV$$

Using vector identity $\nabla . (\emptyset \nabla \emptyset) = \emptyset \nabla^2 \emptyset + (\nabla \emptyset)^2$

$$\int_{S1+S2+S3+\cdots+Sn} \emptyset \nabla \emptyset . dS = \int_{Vo} (\emptyset \nabla^2 \emptyset + (\nabla \emptyset)^2) . dV$$



Uniqueness Theorem

Using Laplace's equation at all points in V0

$$\int_{S1+S2+S3+\cdots+Sn} \emptyset \nabla \emptyset . dS = \int_{Vo} (\nabla \emptyset)^2 . dV$$

Let Φ1 and Φ2 be the solutions for given boundary conditions

$$\int_{S} (\emptyset_1 - \emptyset_2) \nabla (\emptyset_1 - \emptyset_2) . dS = \int_{Vo} \left(\nabla (\emptyset_1 - \emptyset_2)\right)^2 . dV$$

$When, \emptyset_1 = \emptyset_2 \; ; \; \nabla \emptyset_1 = \nabla \emptyset_2$

$$\int_{Vo} \left(\nabla (\emptyset_1 - \emptyset_2)\right)^2 . dV = 0 \qquad \nabla (\emptyset_1 - \emptyset_2) = 0 \qquad \nabla \emptyset_1 = \nabla \emptyset_2$$

$$\emptyset_1 = \emptyset_2 + constant$$

Thus the potentials can differ at most by an additive constant for a given boundary condition which makes no contribution to gradient. Thus for given boundary condition, Laplace equation has unique solution.

Boundary Conditions

- To solve the given Poisson (or Laplace) equation, a suitable boundary condition is required to establish a unique and valued solution inside the bounded region.
- There are two possible boundary conditions,
- **Dirichlet boundary condition** in which the potential on a closed surface S is defined as,

$$\emptyset(r)_{res} = f(r)$$

- **Neumann boundary condition** in which the electric field (normal derivative of potential) everywhere on the surface is defined as,

$$\boldsymbol{n}.\nabla\emptyset(r)_{res} = \frac{\partial\emptyset}{\partial n_{res}} = g(r)$$

- As mentioned in Uniqueness theorem, the solution of Laplace or Poisson equations are unique when they are subjected to the above boundary conditions

Green's Reciprocity Theorem

- Consider a set of n point charges q_i placed at points where the potential due to other charges are given by ϕ_j. The potential at j^{th} point due to charges q_i at other point is

$$\emptyset_j = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ij}}$$

- The potential at j^{th} point due to charges q_i' at other point is

$$\emptyset'_j = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q'_i}{r_{ij}}$$

- Multiplying q_j' by eqn. 1 and q_j by eqn. 2 and summing over index j is

$$\sum_{j=1}^{n}\emptyset_j q'_j = \frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{q_i q'_j}{r_{ij}}$$

$$\sum_{j=1}^{n}\emptyset'_j q_j = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q'_i q_j}{r_{ij}}$$

- Interchanging summation indices

$$\sum_{j=1}^{n}\emptyset_j q'_j = \sum_{j=1}^{n}\emptyset'_j q_j$$

This is Green's reciprocity theorem which is useful to transform the solution of a known problem into the solution of undesired unknown problem.

Green's Reciprocity Theorem

- To handle the boundary conditions it is necessary to develop some new mathematical tools, called as Green's identities or theorems.
- By Gauss's Divergence theorem, $\int \nabla . \vec{\boldsymbol{A}}\, dv = \int \vec{\mathbf{A}} . \hat{\boldsymbol{n}}\, ds$
- If ϕ and ψ are arbitrary scalar fields $\boldsymbol{A} = \Phi \boldsymbol{\nabla} \Psi$ then,

$$\nabla . \boldsymbol{A} = \nabla . (\Phi \nabla \Psi) = \emptyset \nabla^2 \Psi + \nabla \emptyset \nabla \Psi \qquad \vec{\mathbf{A}} . \hat{\boldsymbol{n}} = (\Phi \nabla \Psi) . \hat{\boldsymbol{n}} = \Phi \frac{\partial \Psi}{\partial \mathrm{n}}$$

- Substituting the above in Gauss's divergence theorem gives **Green's first identity**,

$$\int (\emptyset \nabla^2 \Psi + \nabla \emptyset \nabla \Psi)\, dv = \int \Phi \frac{\partial \Psi}{\partial \mathrm{n}}\, ds$$

- Interchanging scalar field and subtracting with above equation gives **Green's second identity**,

$$\int (\Psi \nabla^2 \emptyset + \nabla \Psi \nabla \emptyset)\, dv = \int \Psi \frac{\partial \emptyset}{\partial \mathrm{n}}\, ds \qquad \int (\emptyset \nabla^2 \Psi - \Psi \nabla^2 \emptyset)\, dv = \int \left(\Phi \frac{\partial \Psi}{\partial \mathrm{n}} - \Psi \frac{\partial \emptyset}{\partial \mathrm{n}} \right) ds$$

Formal solution of Potential with Green's Function

- With aid of Green's reciprocity theorem, it can be shown that Green's function for a particular unit charge at point r' and point of observation r is a symmetric function and allow interchangeability

$$\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') = \boldsymbol{G}(\boldsymbol{r}',\boldsymbol{r})$$

$$\Phi(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')\,dv}{|r-r'|} = \int \rho(r')\,G_0(r,r')dv$$

- Let ϕ be the desired solution and ψ=G be the Green's function. Then

$$G(r,r') = \frac{1}{4\pi\varepsilon_0}\frac{1}{r-r'} + F(r,r')$$

Potential due to the induced charge on surface S

$$\nabla^2 G(r,r') = \frac{1}{4\pi\varepsilon_0}\left[\nabla^2\left(\frac{1}{r-r'}\right)\right] + \nabla^2 F(r,r')$$

- Here

$$\nabla^2\left(\frac{1}{r-r'}\right) = -4\pi\delta(r-r') \qquad \nabla^2 F(r,r') = 0 \qquad \nabla^2 G(r,r') = \frac{-\delta(r-r')}{\varepsilon_0}$$

$$\int (\emptyset\nabla^2 G - G\nabla^2\emptyset)\,dv = \int\left(\Phi\frac{\partial G}{\partial \mathrm{n}} - G\frac{\partial \emptyset}{\partial \mathrm{n}}\right)\,ds$$

Formal solution of Potential with Green's Function

$$\int \left(\emptyset(r')\nabla^2 G(r,r') - G(r,r')\nabla^2 \emptyset(r') \right) dv = \int \left(\Phi(r')\nabla G(r,r') - G(r,r')\nabla \Phi(r') \right) ds$$

$$\int \left(-\emptyset(r') \frac{\delta(r-r')}{\varepsilon_0} - G(r,r')\nabla^2 \emptyset(r') \right) dv = \int \left(\Phi(r')\nabla G(r,r') - G(r,r')\nabla \Phi(r') \right) ds$$

$$\frac{-\emptyset(r')}{\varepsilon_0} - \int G(r,r')\nabla^2 \emptyset(r')\, dv = \int \left(\Phi(r')\nabla G(r,r') - G(r,r')\nabla \Phi(r') \right) ds$$

$$\emptyset(r') = -\varepsilon_0 \int G(r,r')\nabla^2 \emptyset(r')\, dv - \varepsilon_0 \int \left(\Phi(r')\nabla G(r,r') - G(r,r')\nabla \Phi(r') \right) ds$$

- Apply Dirichlet boundary condition to ensure the uniqueness of potential on surface S G(r, r') =0, r'on S

$$\emptyset(r) = -\varepsilon_0 \int G(r,r')\nabla^2 \emptyset(r')\, dv - \varepsilon_0 \int \Phi(r')\nabla G(r,r')\, ds$$

Formal solution of Potential with Green's Function

- Case I : The surface surrounding the point r' is grounded

$\Phi(r') = 0$

$$\nabla^2 \emptyset(r') = \frac{-\rho(r')}{\varepsilon_0}$$

$$\emptyset(r) = \int G(r, r')\rho(r')\, dv$$

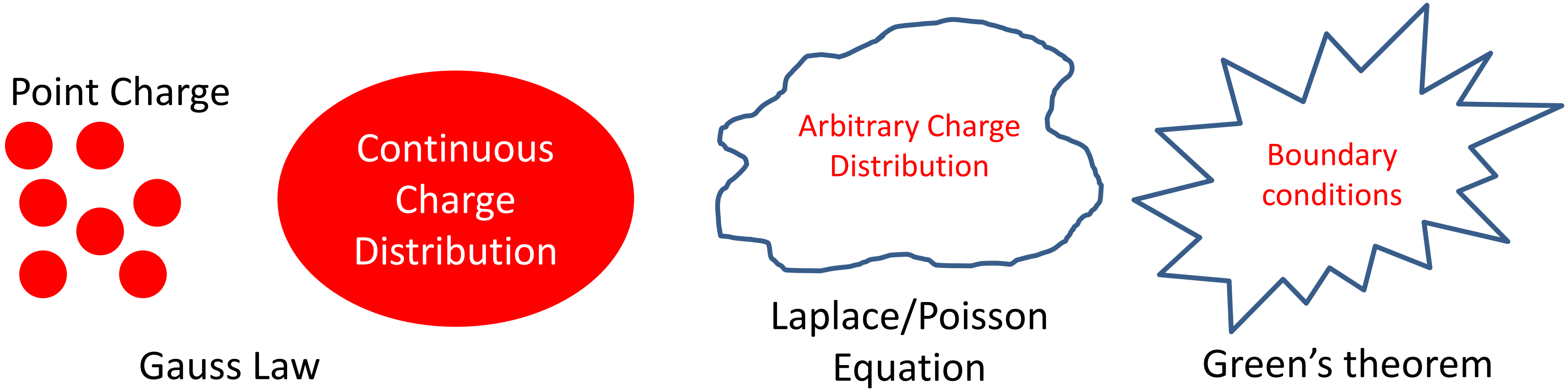
- Case II : When there are no sources of ϕ throughout the volume

$\nabla^2 \emptyset(r') = 0$

$$\emptyset(r) = -\varepsilon_0 \int \Phi(r')\nabla G(r, r')\, ds$$

- In both cases the potential within a region enclosed by a boundary is obtained. In the first case potential is expressed in terms of volume integral and second case potential is expressed in terms of surface integral

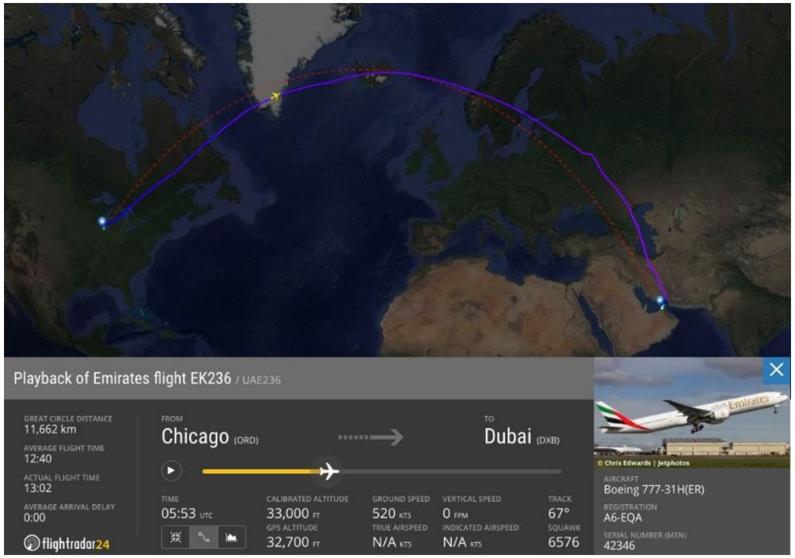
Beyond Green's Theorem



- For most of the electrostatic problems, Green's theorem can be applied as either boundary surface potential or surface charge density is specified.
- However in real situations, if Green's function is difficult to identify, three techniques are available to solve boundary value problems
  - Method of images
  - Expansion in orthogonal function
  - Finite element analysis (FEA)

Method of Images

- Lord Kelvin (1824-1907) invented method of images to solve many special electrostatics problems.
- Complicated charge distribution are replaced by a single or set of point charges without affecting the boundary conditions of the problem.
- It is the process of placing a image charge in place of complicated charge distribution such that their electrical effects for the given boundary conditions remains the same.



Method of Images

- Potential due to n point charges (q1, q2…qn) at any point is

$$\emptyset = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

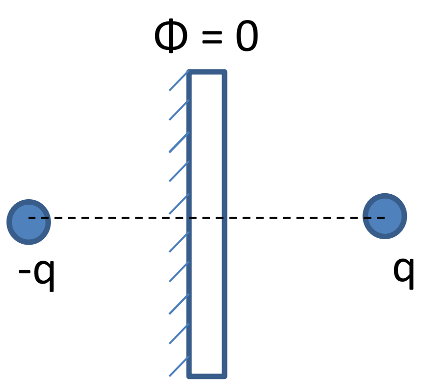
- Potential due to surface of zero potential

$$\emptyset = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} = 0$$

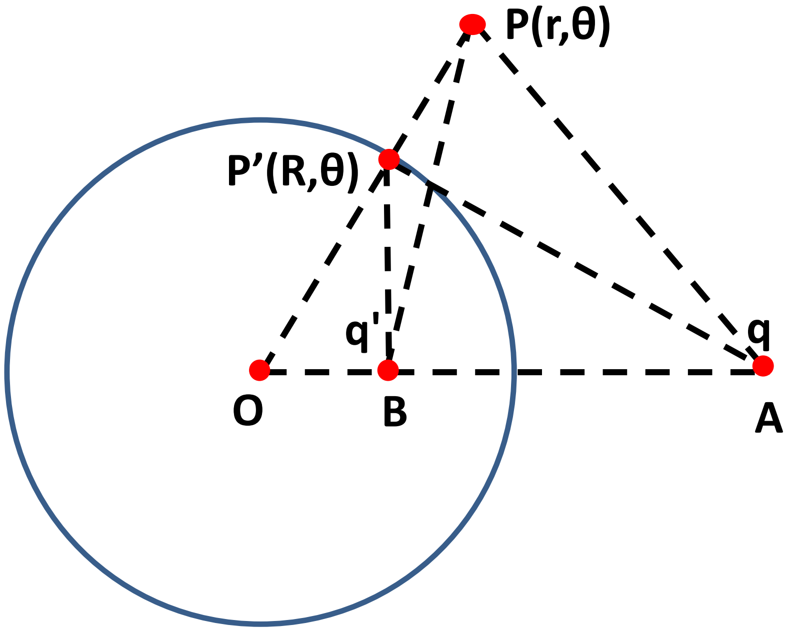
- System of charges q1, q2…qj and grounded conductor which was replaced by system of image charges qj+1, qj+2…qn

$$\emptyset = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{j}\frac{q_i}{r_i} + \frac{1}{4\pi\varepsilon_0}\sum_{i=j+1}^{n}\frac{q_i}{r_i}$$

- A point charge placed in front of a conducting mirror having zero potential



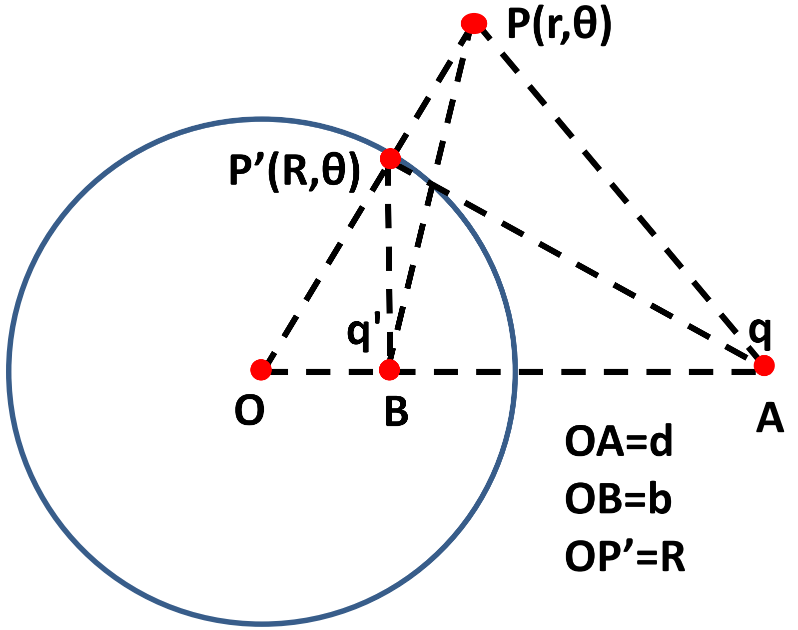
A point charge in front of a conducting sphere which is grounded



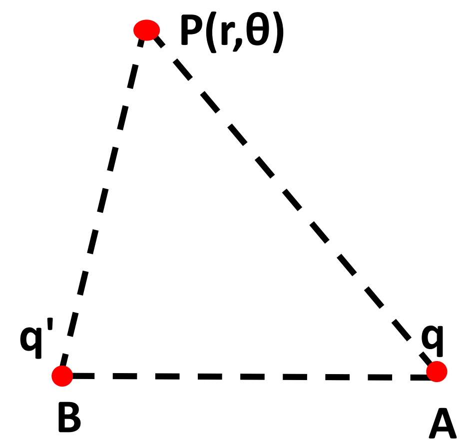
OA=d
OB=b
OP'=R

- Boundary Conditions
  - Electric potential at surface is zero. $\Phi = 0,\ r = R$
  - Electric potential at infinity is zero. $\Phi = 0,\ r = \infty$
- Consider a image charge (q') placed at B such that it satisfies B.C.

A point charge in front of a conducting sphere which is grounded



- Boundary Conditions
  - Electric potential at surface is zero. $\Phi = 0, r = R$
  - Electric potential at infinity is zero. $\Phi = 0, r = \infty$
- Consider a image charge (q') placed at B such that it satisfies B.C.
- Potential due to real and image charge at P' is

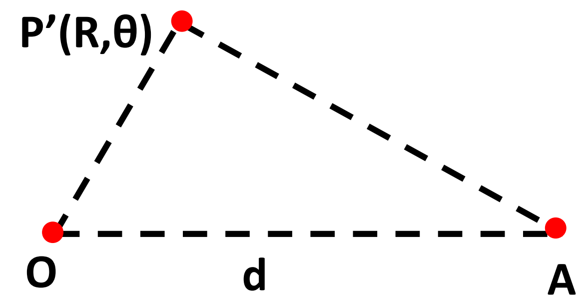


$$\emptyset(P') = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AP'} + \frac{q'}{BP'}\right] = 0$$

$$q' = -q\frac{BP'}{AP'}$$

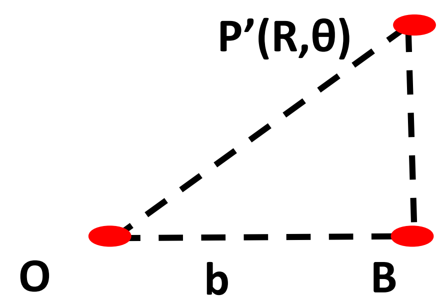
From congruent triangles, $\Delta OP'B \approx \Delta OP'A, \frac{OB}{OP'} = \frac{OP'}{OA}, OB = \frac{R^2}{d}$

A point charge in front of a conducting sphere which is grounded



$$In\ \Delta OAP', (AP')^2 = (OP')^2 + (OA)^2 - 2OP'.OA\cos\theta$$

$$AP' = \sqrt{R^2 + d^2 - 2Rd\cos\theta}$$



$$In\ \Delta OBP', (BP')^2 = (OP')^2 + (OB)^2 - 2OP'.OB\cos\theta$$

$$BP' = \sqrt{R^2 + b^2 - 2Rb\cos\theta}$$

$$q' = -q\frac{\sqrt{R^2 + b^2 - 2Rb\cos\theta}}{\sqrt{R^2 + d^2 - 2Rd\cos\theta}}$$

$$q' = -q\frac{\sqrt{R^2 + \left(\frac{R^2}{d}\right)^2 - 2R\frac{R^2}{d}\cos\theta}}{\sqrt{R^2 + d^2 - 2Rd\cos\theta}}$$

$$q' = -q\left(\frac{R}{d}\right)$$

Image charge is placed at a distance of $b = \frac{R^2}{d}$ from centre of spherical conductor along line joining the centre of sphere and real point charge.

A point charge in front of a conducting sphere which is grounded

Potential at point P due to point charge +q placed near grounded conducting sphere is

$$\emptyset(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AP} + \frac{q'}{BP}\right] \qquad \emptyset(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} + \frac{q'}{\sqrt{r^2 + b^2 - 2rb\cos\theta}}\right]$$

$$\emptyset(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} - \frac{q}{\sqrt{\frac{r^2d^2}{R^2} + R^2 - 2rd\cos\theta}}\right]$$

The component of electric field at P is $\quad E_r = \frac{-\partial\emptyset}{\partial r} \qquad E_\theta = \frac{-\partial\emptyset}{r.\partial\theta}$

$$E_r = \frac{-\partial\emptyset}{\partial r} = \frac{q}{4\pi\varepsilon_0}\left[\frac{r - d\cos\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}} - \frac{r\frac{d^2}{R^2} - d\cos\theta}{\left(\frac{r^2d^2}{R^2} + R^2 - 2rd\cos\theta\right)^{3/2}}\right]$$

$$E_\theta = \frac{-\partial\emptyset}{r.\partial\theta} = \frac{q}{4\pi\varepsilon_0}\left[\frac{d\sin\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}} - \frac{d\sin\theta}{\left(\frac{r^2d^2}{R^2} + R^2 - 2rd\cos\theta\right)^{3/2}}\right]$$

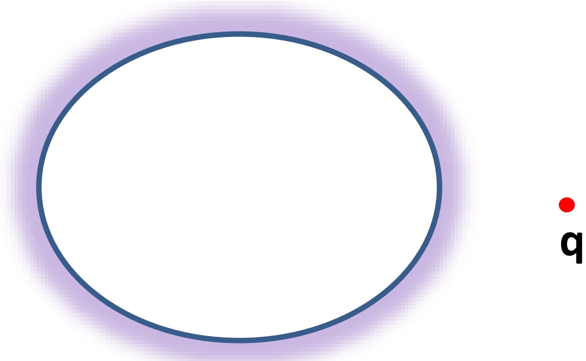
A point charge in front of a conducting sphere which is grounded

Electric field at the surface of sphere is

$$E_r = \left(\frac{-\partial \emptyset}{\partial r}\right)_{r=R} = \frac{q}{4\pi\varepsilon_0 R}\left[\frac{R^2 - d^2}{(R^2 + d^2 - 2Rd\cos\theta)^{3/2}}\right] = 0$$

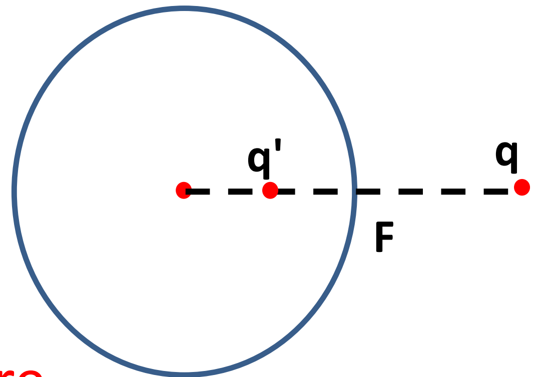
Surface charge density is

$$\sigma = \varepsilon_0 (E_r)_{r=R} = \frac{q}{4\pi R}\left[\frac{R^2 - d^2}{(R^2 + d^2 - 2Rd\cos\theta)^{3/2}}\right]$$



The force between the sphere and point charge is

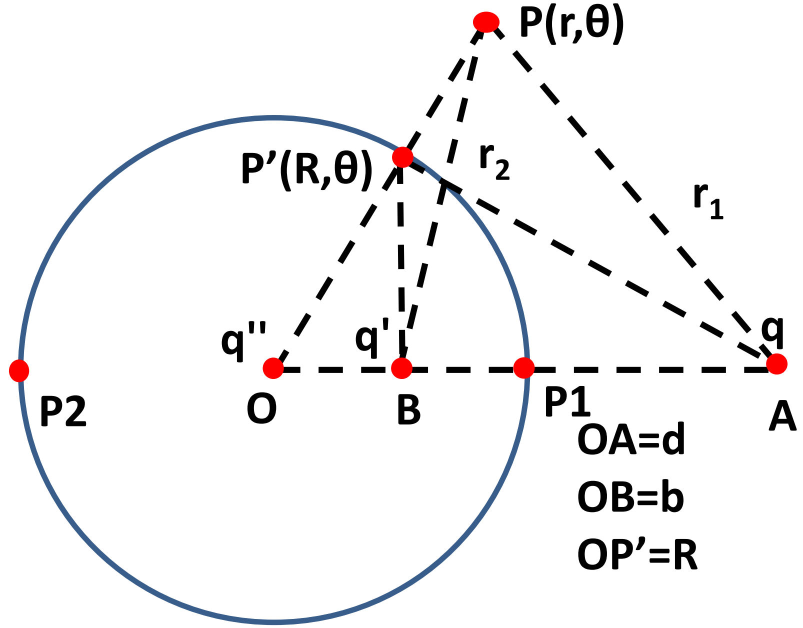
$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{(AB)^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2 Rd}{(d^2 - R^2)^2}$$



Negative sign indicate the force is attractive in nature.

A point charge in front of a conducting sphere which is insulated

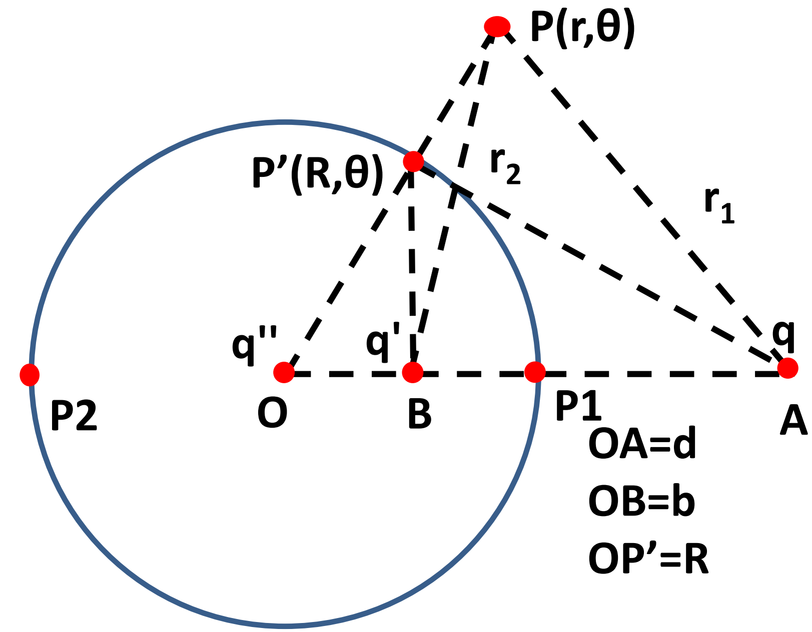
- Boundary Conditions
  - Electric potential at surface is zero. Φ = 0, r = R
  - Electric potential at infinity is zero. Φ = 0, r = ∞
  - Potential on sphere is uniform throughout.
  - Net charge on conductor remains zero.



Consider a image charge (q') placed at B such that it satisfies first and second boundary conditions.

To satisfy the remaining boundary conditions, consider a charge (q") at centre of sphere so that it provides zero net charge and keeps the potential constant.

A point charge in front of a conducting sphere which is insulated



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  - Electric potential at surface is zero. $\Phi = 0$, $r = R$
  - Electric potential at infinity is zero. $\Phi = 0$, $r = \infty$
  - Potential on sphere is uniform throughout.
  - Net charge on conductor remains zero.

The potential on the spherical surface is $\phi = \dfrac{qR/d}{4\pi\varepsilon_0 R} = \dfrac{q}{4\pi\varepsilon_0 d}$

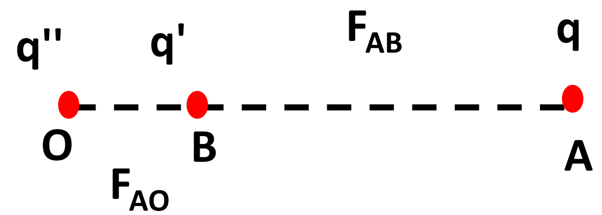
The potential at point P due to combination of point charge +q and insulated charged sphere is

$$\phi(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} + \frac{q'}{r_2} + \frac{q''}{r}\right] \qquad q' = \frac{-qR}{d} \qquad q'' = \frac{qR}{d}$$

A point charge in front of a conducting sphere which is insulated

Force of attraction between the conducting sphere due to induced charge and the point charge q must be the resultant of the force between q and q’ at B and q and q’’ at O.

$$F = \frac{-1}{4\pi\varepsilon_0}\frac{q.\frac{qR}{d}}{\left(d-\frac{R^2}{d}\right)^2} + \frac{1}{4\pi\varepsilon_0}\frac{q.\frac{qR}{d}}{d^2}$$



$$F = \frac{-1}{4\pi\varepsilon_0}\frac{q^2R}{d^3}\left(\frac{1}{\left(1-\frac{R^2}{d^2}\right)}-1\right)$$

Surface charge density at P’ due to q and q’ is

$$\sigma_1 = \frac{-q(d^2-r^2)}{4\pi R(R^2+d^2-2Rd\cos\theta)^{3/2}}$$

Surface charge density at P’ due to q’’ is

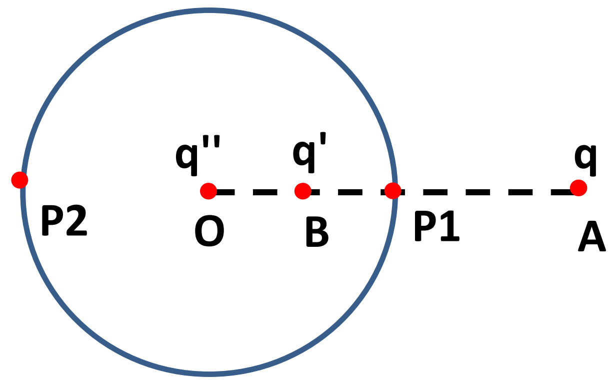
$$\sigma_2 = \frac{qR/d}{4\pi R^2}$$

$$\sigma = \sigma_1 + \sigma_2 = \frac{-q(d^2-R^2)}{4\pi R(R^2+d^2-2Rd\cos\theta)^{3/2}} + \frac{q}{4\pi Rd}$$

A point charge in front of a conducting sphere which is insulated

At P_1 nearest to q is [θ= 0°]

$$\sigma_{P_1} = \frac{-q(d^2 - R^2)}{4\pi R(R^2 + d^2 - 2Rd)^{3/2}} + \frac{q}{4\pi Rd}$$



$$\left(\sigma_{P_1}\right)_{\theta=0^o} = \left(\frac{d^2 - R^2}{(d-R)^3} - \frac{1}{d}\right)\frac{-q}{4\pi R} = \frac{-q(3d - R)}{4\pi d(d-R)^2}$$

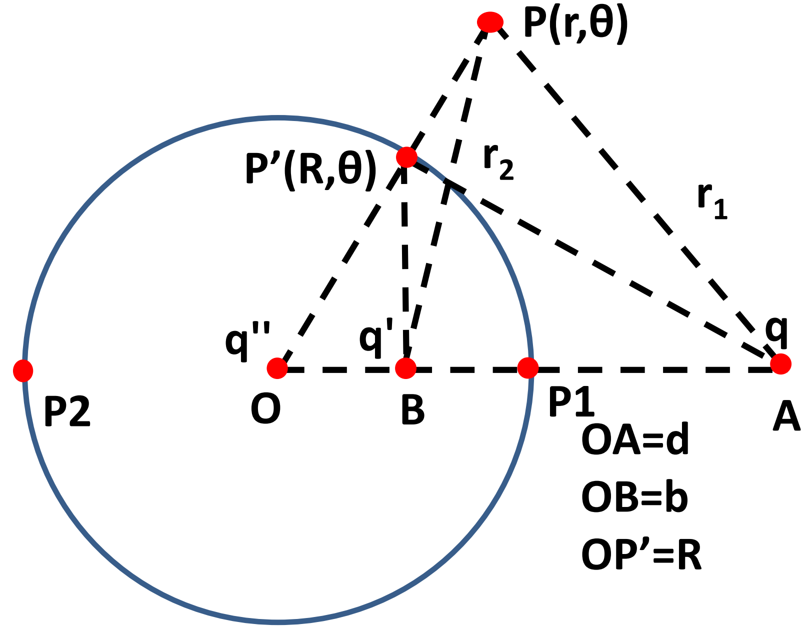
At P_2 farthest to q is [θ= 180°]

$$\left(\sigma_{P_2}\right)_{\theta=180^o} = \frac{q(3d + R)}{4\pi d(d+R)^2}$$

Surface charge density at nearest and farthest is negative and positive. There must be a place where surface charge density is zero, called as circle of no electrification.

A point charge in front of a conducting sphere which is charged and insulated

- Boundary Conditions
  - Potential on sphere is uniform throughout.
  - Net charge on conductor remains +e.
  - Electric potential at infinity is zero. $\Phi = 0, r = \infty$
  - $\nabla^2 \emptyset = 0$ in external space expect A



Consider a image charge (q') placed at B such that it satisfies first and second boundary conditions.

To satisfy the remaining boundary conditions, consider a charge (q") at centre of sphere so that it provides +e charge.

A point charge in front of a conducting sphere which is charged and insulated

- Boundary Conditions
  - Potential on sphere is uniform throughout.
  - Net charge on conductor remains +e.
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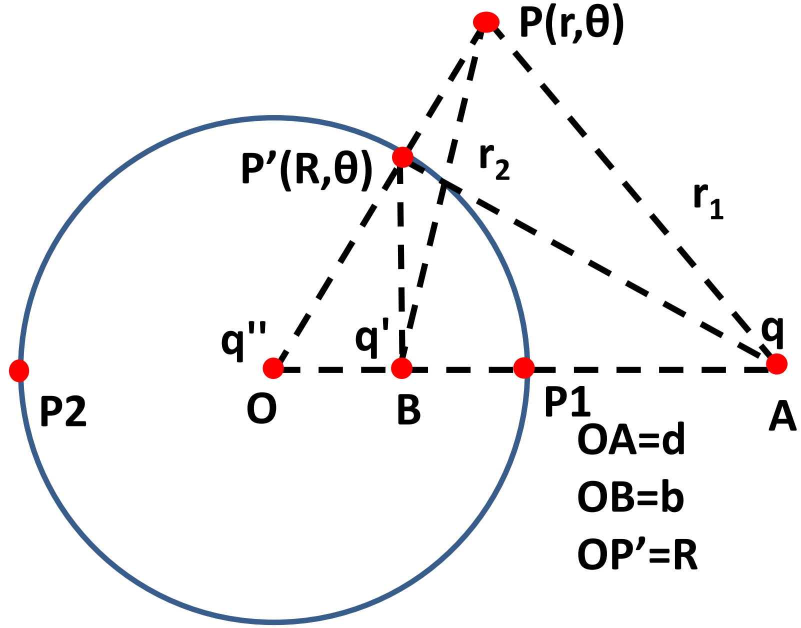


The potential on the spherical surface is $\emptyset = \dfrac{e + {}^{qR}/_{d}}{4\pi\varepsilon_0 R} = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{e}{R} + \dfrac{q}{d}\right)$

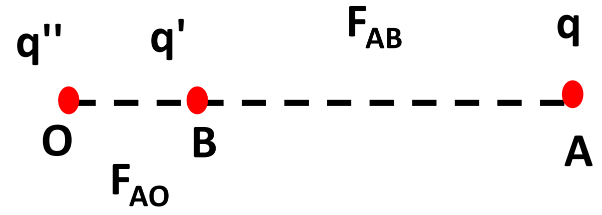
The potential at point P due to combination of point charge +q and charged and insulated charged sphere is

$$\emptyset(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} + \frac{q'}{r_2} + \frac{q''}{r}\right] \qquad q' = \frac{-qR}{d} \qquad q'' = e + \frac{qR}{d}$$

A point charge in front of a conducting sphere which is insulated and charged

Force of repulsion between the conducting sphere due to induced charge and the point charge q must be the resultant of the force between q and q' at B and q and q'' at O.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{-q.\frac{qR}{d}}{\left(d-\frac{R^2}{d}\right)^2} + \frac{1}{4\pi\varepsilon_0}\frac{q.\left(e+\frac{qR}{d}\right)}{d^2}$$



$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{eq}{d^2} + \frac{q^2R}{d^3} - \frac{q^2Rd}{(d^2-R^2)^2}\right]$$

If A is very near to spherical surface, put d = R + x

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{eq}{(R+x)^2} + \frac{q^2R}{(R+x)^3} - \frac{q^2Rd}{\left((R+x)^2-R^2\right)^2}\right]$$

Here when x is negligibly small

$$(R+x)^2 = R^2; (R+x)^3 = R^3; \left((R+x)^2 - R^2\right)^2 = (2R+x)^2x^2$$

A point charge in front of a conducting sphere which is insulated and charged

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{eq}{R^2} + \frac{q^2 R}{R^3} - \frac{q^2 Rd}{(2R+x)^2 x^2}\right]$$

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{q(q+e)}{R^2} - \frac{q^2}{4x^2}\right]$$

For the force to be repulsive, F must be positive.

$$\frac{q(q+e)}{R^2} > \frac{q^2}{4x^2} \qquad e > q\left[\frac{R^2}{4x^2} - 1\right] \qquad e > \frac{qR^2}{4x^2} \qquad x > \frac{R}{2}\sqrt{\frac{q}{e}}$$

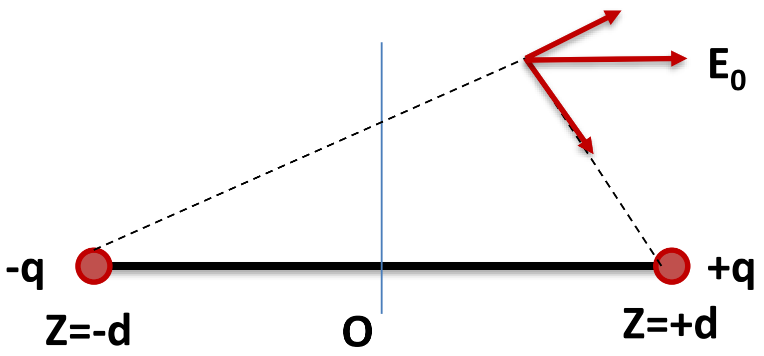
Surface charge density at P' due to q and q' and q" is

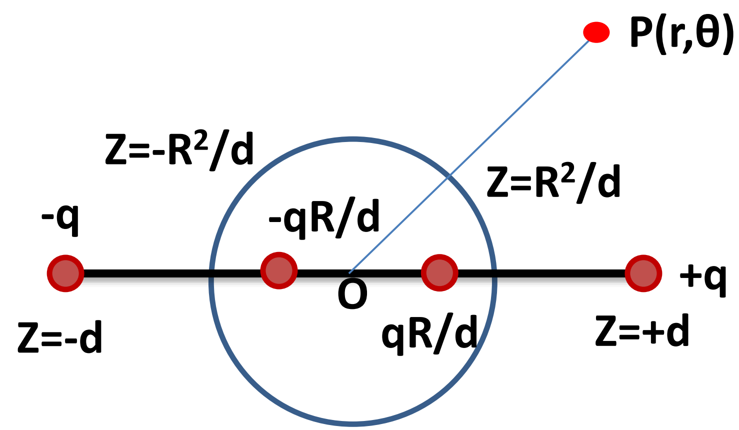
$$\sigma = \frac{-q(d^2 - R^2)}{4\pi R(R^2 + d^2 - 2Rd\cos\theta)^{3/2}} + \frac{\left(e + \frac{qR}{d}\right)}{4\pi R^2}$$

$$\left(\sigma_{P_1}\right)_{\theta=0^o} = \frac{-q(d^2 - R^2)}{4\pi R(R^2 + d^2 - 2Rd)^{3/2}} + \frac{\left(e + \frac{qR}{d}\right)}{4\pi R^2} = \frac{-q(d-R)}{4\pi R(d-R)^2} + \frac{e}{4\pi R^2}$$

$$\left(\sigma_{P_2}\right)_{\theta=180^o} = \frac{-q(d^2 - R^2)}{4\pi R(R^2 + d^2 + 2Rd)^{3/2}} + \frac{\left(e + \frac{qR}{d}\right)}{4\pi R^2} = \frac{q(3d+R)}{4\pi R(d+R)^2} + \frac{e}{4\pi R^2}$$

Conducting sphere in a uniform electric field





- Consider a conducting sphere of radius R in a uniform electric field E_0. A uniform electric filed can be produced by positive and negative charges at infinity.
- Potential at any field due to charge +q,-q and their images is

$$\emptyset(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{r^2+d^2+2rd\cos\theta}} - \frac{q}{\sqrt{r^2+d^2-2rd\cos\theta}} - \frac{\left({}^{qR}/_{d}\right)}{\sqrt{r^2+\frac{R^4}{d^2}+\frac{2R^2r}{d}\cos\theta}} + \frac{\left({}^{qR}/_{d}\right)}{\sqrt{r^2+\frac{R^4}{d^2}-\frac{2R^2r}{d}\cos\theta}}\right]$$

Conducting sphere in a uniform electric field

$$\frac{q}{\sqrt{r^2 + d^2 + 2rd\cos\theta}} = \frac{q}{d}\left[1 + \frac{r^2}{d^2} + \frac{2r}{d}\cos\theta\right]^{-1/2} = \frac{q}{d}\left[1 - \frac{r}{d}\cos\theta\right]$$

$$\frac{q}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} = \frac{q}{d}\left[1 + \frac{r}{d}\cos\theta\right]$$

$$\frac{\left({}^{qR}/_{d}\right)}{\sqrt{r^2 + \frac{R^4}{d^2} + \frac{2R^2 r}{d}\cos\theta}} = \frac{qR}{d}\left[1 - \frac{R^2}{rd}\cos\theta\right]$$

$$\frac{\left({}^{qR}/_{d}\right)}{\sqrt{r^2 + \frac{R^4}{d^2} - \frac{2R^2 r}{d}\cos\theta}} = \frac{qR}{d}\left[1 + \frac{R^2}{rd}\cos\theta\right]$$

Conducting sphere in a uniform electric field

Potential at P due to point charge and sphere is

$$\emptyset(r,\theta) = \frac{1}{4\pi\varepsilon_0}\left[\frac{-2q}{d^2} r\cos\theta + \frac{2q}{d^2}\frac{R^3}{r^2}\cos\theta\right]$$

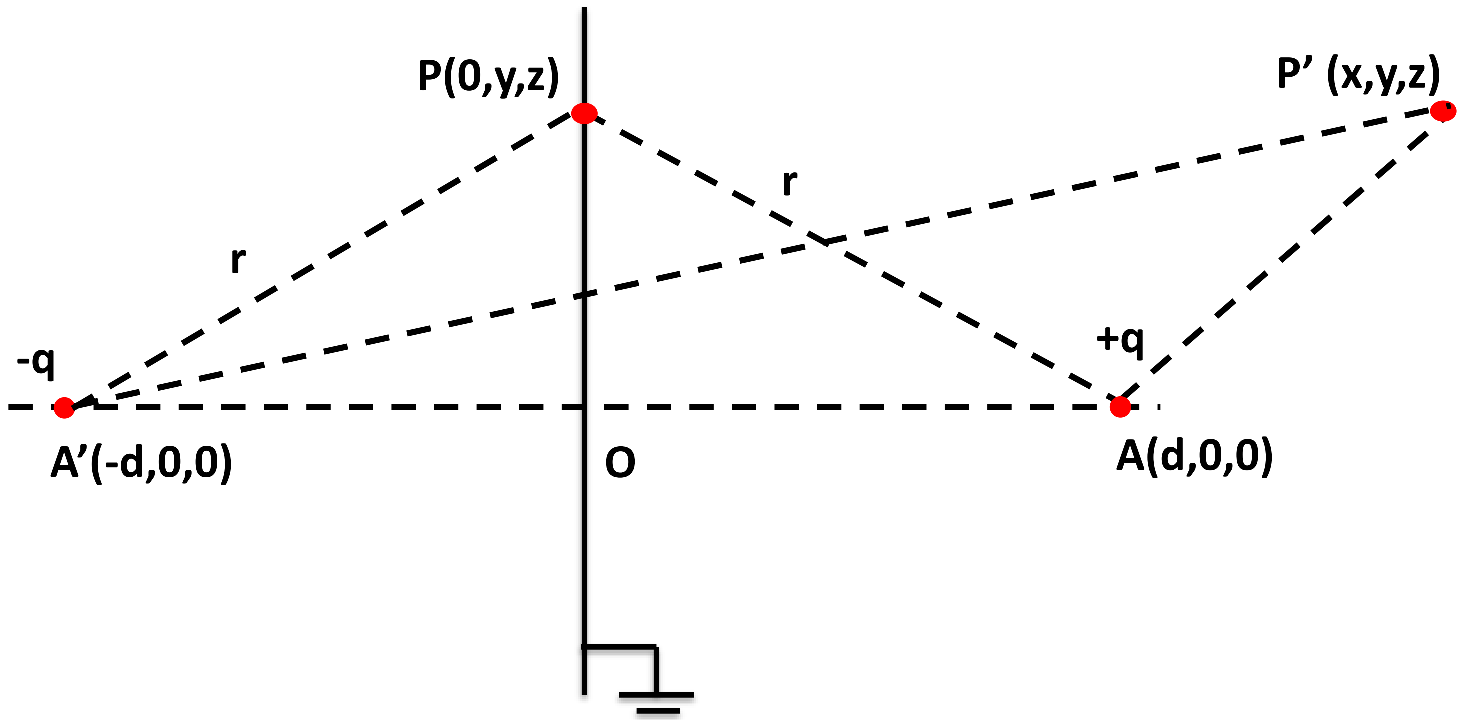
$$\emptyset(r,\theta) = -E_0\left(r - \frac{R^3}{r^2}\right)\cos\theta$$

Potential due to uniform electric field and induced surface charge density, as Z= r cos θ

$$\emptyset(P) = -E_0 z + E_0\frac{R^3 z}{r^3}$$

$$\sigma = \varepsilon_0 (E_r)_{r=R} = -\varepsilon_0 \frac{\partial \emptyset}{\partial r} == -\varepsilon_0 E_0 \cos\theta$$

A point charge near an infinite grounded conducting plane



- Boundary Conditions
  - Electric potential at surface is zero. $\Phi = 0$, $x = 0$
  - Electric potential at infinity is zero. $\Phi = 0$, $x = \infty$

Books for Reference

1. **J. D. Jackson**, *Classical Electrodynamics* (Wiley Eastern Ltd., New Delhi, 1999)
2. **D. Griffiths**, *Introduction to Electrodynamics* (Prentice-Hall of India, New Delhi, 1999)
3. **R. P. Feynman, R. B. Leighton and M. Sands**, *The Feynman Lectures on Physics: Vol. II (*Narosa Book Distributors, New Delhi, 1989)
4. **Satya Prakash**, *Electromagnetic Theory and Electrodynamics* (Kedar Nath Ram Nath, Meerut, 2015)