Bharathidasan University

Tiruchirappalli – 620 024, Tamil Nadu, India

Programme: M. Sc., Physics

Course Title : Electromagnetic Theory

Course Code : 22PH301

Unit IV
Introduction to Electrodynamics

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Electromotive Force and Ohm's Law

For a current to flow, charges has to be pushed. For any substance, if σ is conductivity and f is force per unit charge and ρ is resistivity, then

$$J = \sigma f$$

The electromagnetic force that drives charges is,

$$J = \sigma(E + vXB)$$

Since the velocity of charges is sufficiently small, ignore the second term and it gives Ohm's law

$$J = \sigma E$$

It states, total current is proportional to the potential difference,

$$V = IR \ and \ R = {}^{L}/_{\sigma A}$$

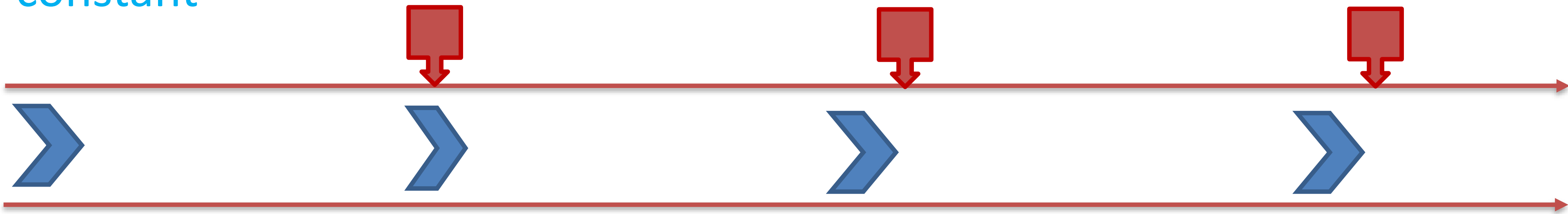
For steady current and uniform conductivity,

$$\nabla . E = \frac{1}{\sigma} \nabla . J = 0$$

Electromotive Force and Ohm's Law

In a given electric field **E**, according to Newton's II law, charge should accelerate due to the force qE produced. Hence current increases with time. This is in contrast with Ohm's law, which states a constant field produces a constant current

To remove the contradiction, if the frequent collision of electron's is taken in account. For example, while driving in a street with stop signs, acceleration occur between stops, while average speed remains constant



Also density of moving charges decreases with increase in temperature. So work done by electrical force is converted into heat as a result of collision. The power delivered is given by Joule's law of heating as,

$$P = VI = I^2R$$

Electromotive Force and Ohm's Law

- In a circuit, the current is same around the loop. Why?
- It can be expected to have large charges in battery and none in lamp. What is the pushing factor?

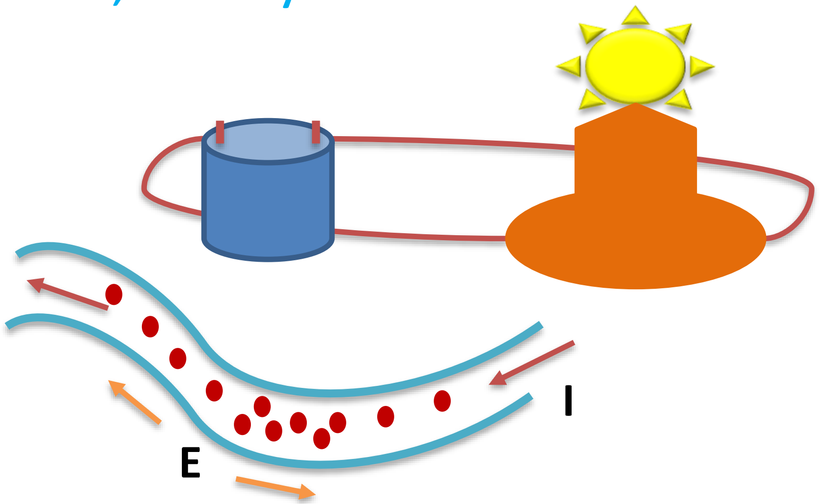
Consider a current in a bend. When the charges piles up in the knee, it produces a field that opposes the current flowing in and promote the current flowing out. So for no accumulation, the system acts like a self-correcting automatic system.

If f_s is source, then electrostatic force is

$$f = f_s + E$$

The net effect around circuit is emf,

$$\epsilon = \oint f.dl = \oint f_s.dl$$



Electromotive force (emf) in a conducting circuit is the total accumulated force on the charges throughout the length of the loop. It is nothing but the potential, V

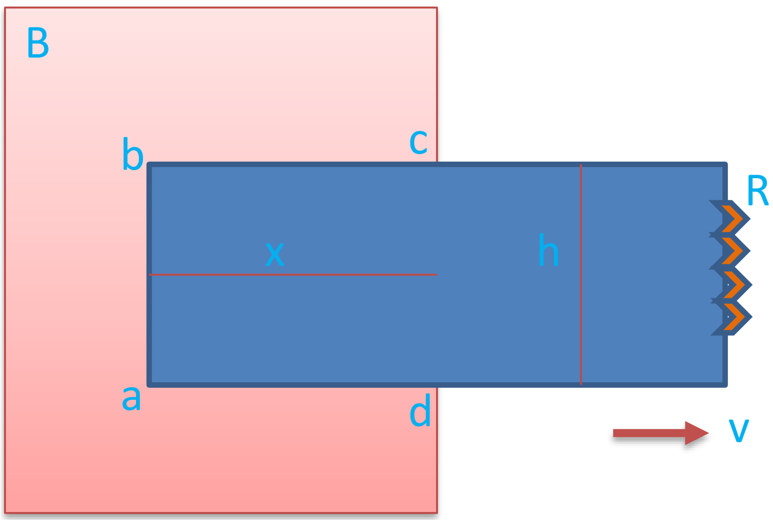
Electromotive Force and Ohm's Law

From a uniform magnetic field, if a loop is pulled to right with velocity (v), charges in section 'ab' experiences a magnetic force. Then its emf is,

$$\epsilon = \oint f_{mag}.dl = vBh$$

Although the magnetic force is establishing the emf, it is not doing work. It is the one who pulls the loop, f_{pull} = vB. Let Φ be the flux of B through the loop,

$$\emptyset = \int B.da$$



As the loop moves, flux decreases

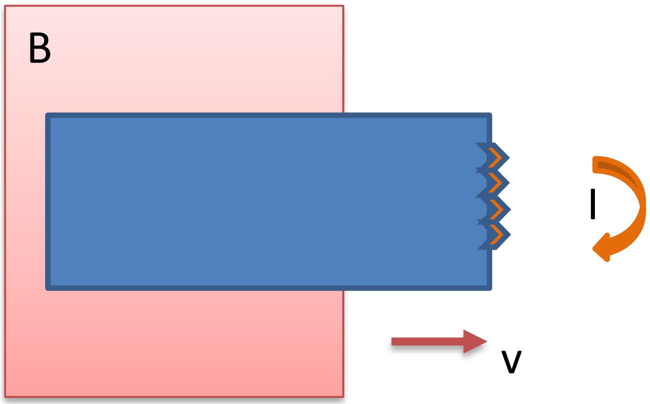
$$\frac{d\emptyset}{dt} = Bh\frac{dx}{dt} = -Bhv$$

$$\epsilon = -\frac{d\emptyset}{dt}$$

emf generated is minus rate of change of flux which is called as "flux rule" for motional emf.

Faraday's Law of Induction

In 1831, Faraday reported series of experiment to demonstrate electromagnetic induction.



Exp. 1: Pull a loop of wire to right through a magnetic field

Faraday's Law of Induction

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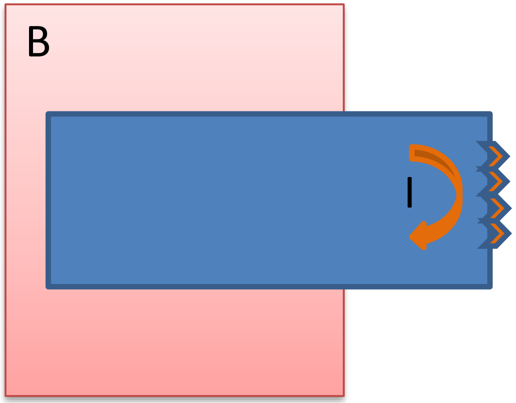

Exp. 2: Move a magnet to left holding the loop of wire still

Faraday's Law of Induction

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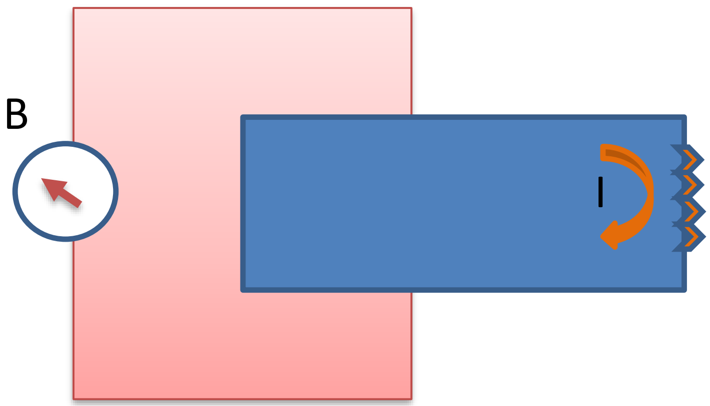

Exp. 3: Keep magnet and loop at rest and change the magnetic field

Faraday's Law of Induction

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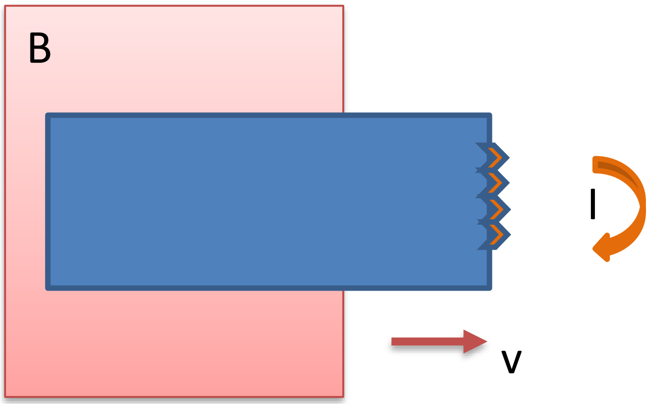
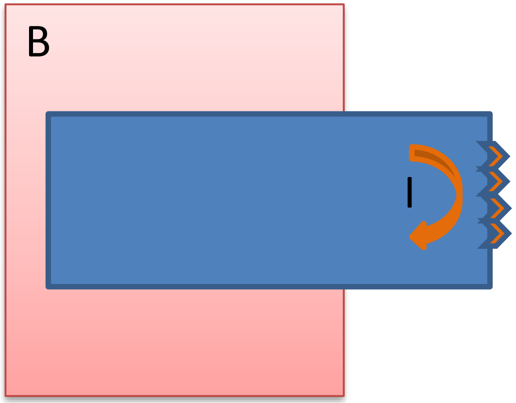
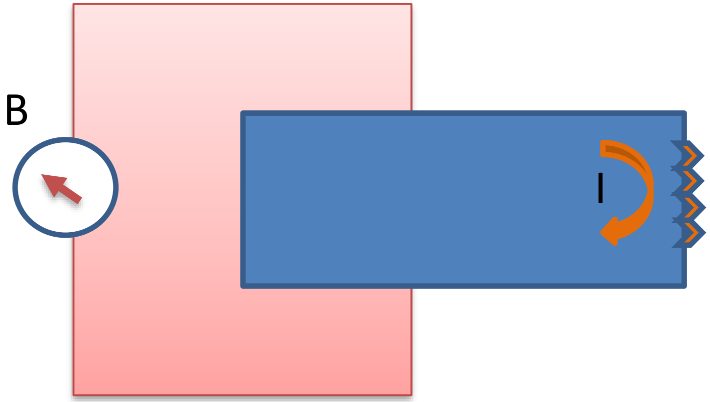




Exp. 1: Pull a loop of wire to right through a magnetic field

Exp. 2: Move a magnet to left holding the loop of wire still



Exp. 3: Keep magnet and loop at rest and change the magnetic field

In all the experiments, current flows in the loop. "a changing magnetic field induces an electric field"

Faraday's Law of Induction

Faraday law states, "Whenever the magnetic flux linked with a closed circuit changes, an induced emf is setup in the circuit whose magnitude at any instant is proportional to the rate of change of magnetic flux linked with the circuit."

$$\epsilon \propto \frac{d\emptyset}{dt}$$

Lenz's law states, "The direction of induced emf is such that it opposes the change in the flux that produces it."

$$\epsilon = -\frac{d\emptyset}{dt}$$

"nature abhors a change in flux"

The Faraday's law in integral form is

$$\epsilon = \oint E.dl = -\frac{d\emptyset}{dt} \qquad \oint E.dl = -\int \frac{d\emptyset}{dt} da$$

Applying Stokes theorem

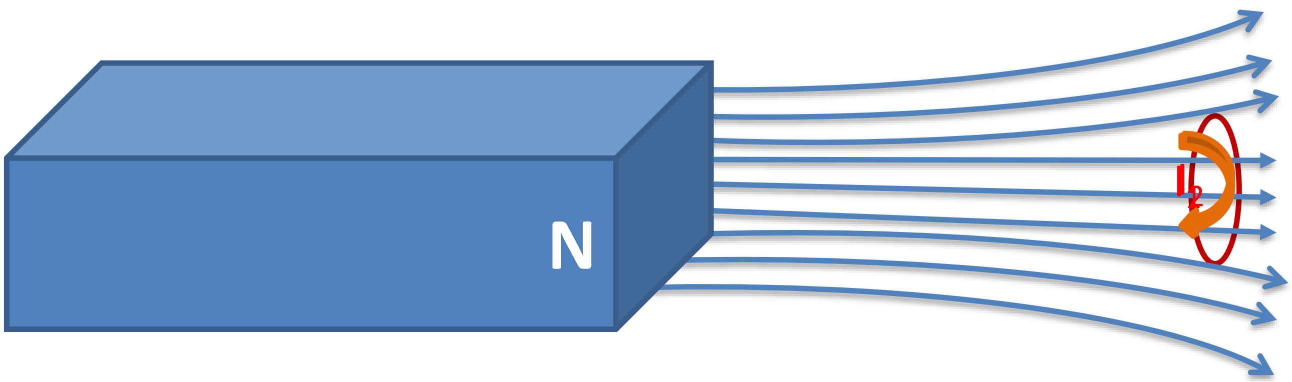
$$\nabla \, X \, \boldsymbol{E} = -\frac{\partial B}{\partial t}$$

Faraday’s Law of Induction

In the coil and magnet experiment, a definite magnetic flux is linked with the coil. If the north pole is moved towards/away, flux through coil increases/decreases, which induces an emf.

As the speed of magnet increases, lines of force increases which induced emf.

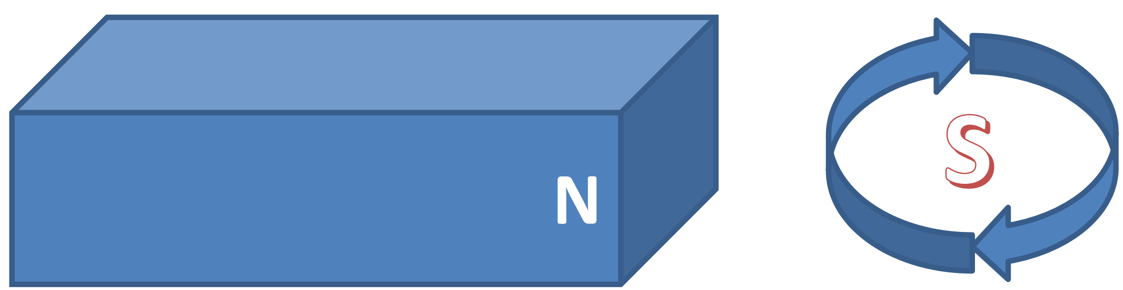
As the speed of magnet decreases, rate of change of flux decreases and induced emf decreases. So induced emf is always proportional to rate of change of flux.



Faraday's Law of Induction

When the north pole is moved towards coil, induced current opposes the motion of magnet. This is possible when nearer face of coil acts like a magnetic north pole which requires anticlockwise current.

When magnet is moved away, nearer face acts like a south pole through clockwise current. This in turn again oppose the motion of magnet. This is Lenz's law and validates law of conservation of energy.



Inductance : Mutual Inductance

Consider a coil-1 consist of conducting wire in the form a solenoid. Around this coil-2 is wound through insulation. Varying current through coil-1 creates a magnetic flux which passes through coil-2 and thus induces emf in coil-2.

The magnetic field inside a solenoid is

$$B = \frac{1}{\varepsilon_0 C^2}\frac{N_1 I_1}{l}$$

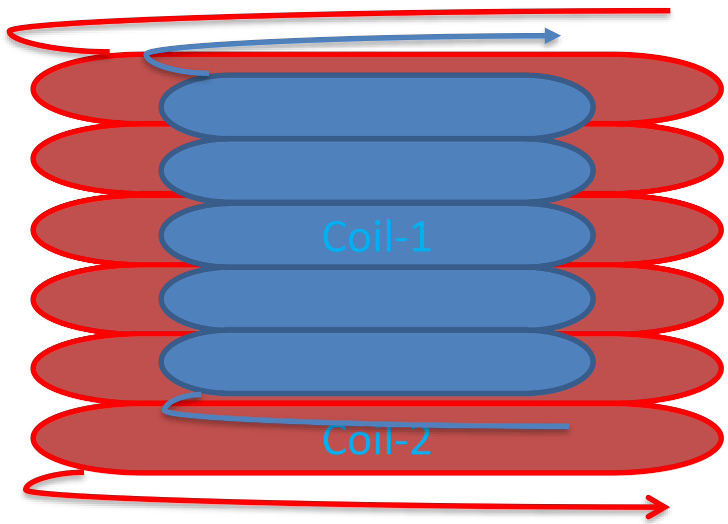
Induced emf in coil-2 is

$$\epsilon_2 = -N_2 S\frac{dB}{dt}$$

$$\epsilon_2 = -\frac{N_1 N_2 S}{\varepsilon_0 C^2 l}\frac{dI_1}{dt} = m_{21}\frac{dI_1}{dt}$$

Induced emf in coil-1 is

$$\epsilon_1 = m_{12}\frac{dI_2}{dt}$$



N_1- number of turns in coil-1

l – length of coil-1

I_1 – current in coil-1

N_2- number of turns in coil-2

I_2 – current in coil-2

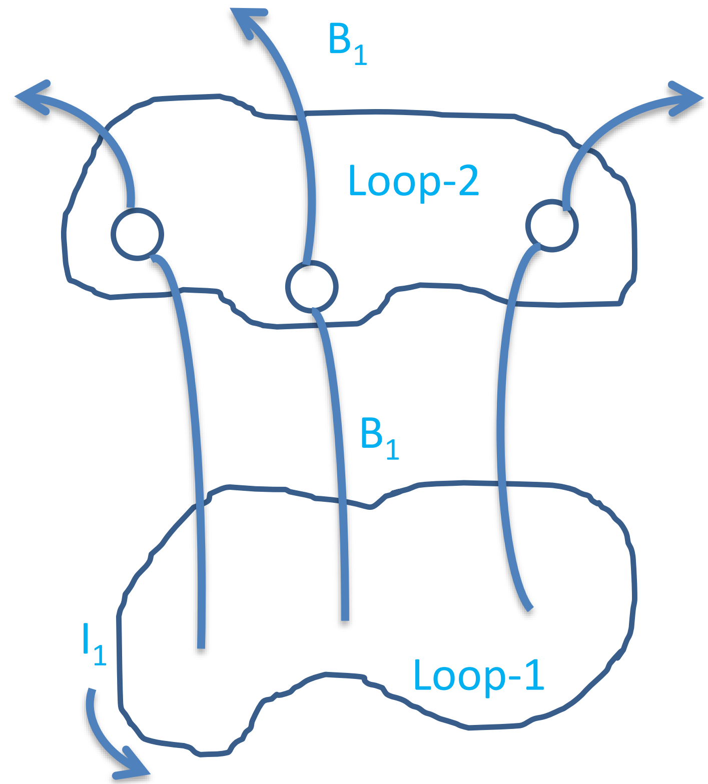
S- cross-section of coil-1

Inductance : Mutual Inductance

According to Biot-Savart law

$$\boldsymbol{B}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\boldsymbol{l}_1 X \boldsymbol{r}}{r^2}$$

Flux through loop-2 is



$$\emptyset_2 = \int B_1 . da_2 = \int (\nabla\ X\ A_1). da_2 = \oint A_1 . dl_2$$

$$\boldsymbol{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\boldsymbol{l}_1}{r}$$

$$\emptyset_2 = \frac{\mu_0 I_1}{4\pi} \oint \oint \frac{d\boldsymbol{l}_1}{r} d\boldsymbol{l}_2$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\boldsymbol{l}_1 . d\boldsymbol{l}_2}{r}$$

M_{21} is mutual inductance

This is Neumann formula. Here M_{21} is purely geometrical quantity that depends upon size, shape and relative position of two loops.

When the role of loop-1 and loop-2 are switched over $m_{21} = m_{12} = M$

Books for Reference

1. **J. D. Jackson**, *Classical Electrodynamics* (Wiley Eastern Ltd., New Delhi, 1999)
2. **D. Griffiths**, *Introduction to Electrodynamics* (Prentice-Hall of India, New Delhi, 1999)
3. **R. P. Feynman, R. B. Leighton and M. Sands**, *The Feynman Lectures on Physics: Vol. II (*Narosa Book Distributors, New Delhi, 1989)
4. **Satya Prakash**, *Electromagnetic Theory and Electrodynamics* (Kedar Nath Ram Nath, Meerut, 2015)