Bharathidasan University
Tiruchirappalli – 620 024, Tamil Nadu, India

Programme: M. Sc., Physics

Course Title : Lasers and Nonlinear Optics
Course Code : 22PH401

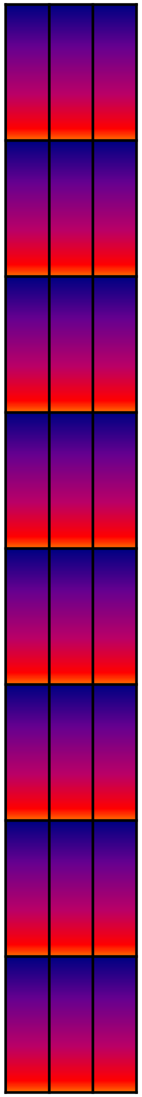
Unit III
Basics of Nonlinear Optics

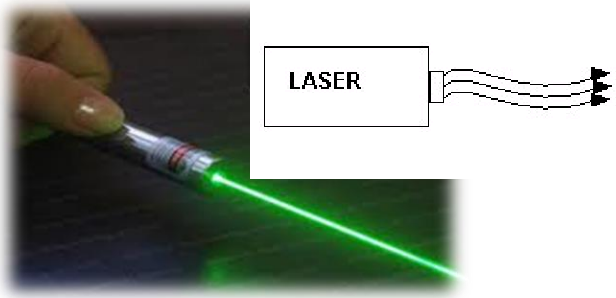
Dr. T.C. Sabari Girisun
Assistant Professor
Department of Physics

Linearity gives BEAUTY
while
Nonlinearity gives EXCITMENT

Lasers : New Vision of Optics

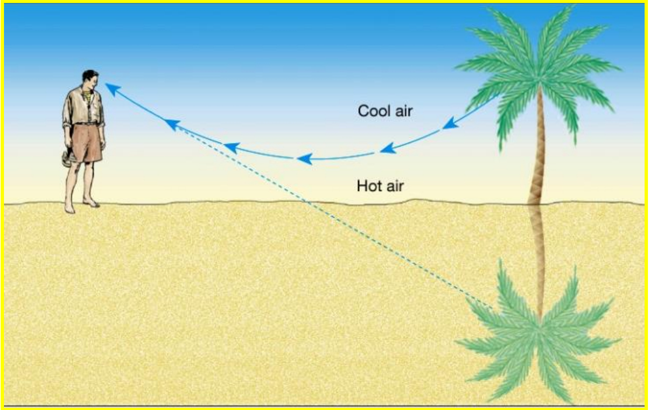
1. **Many wavelengths**
2. **Multidirectional**
3. **Incoherent**

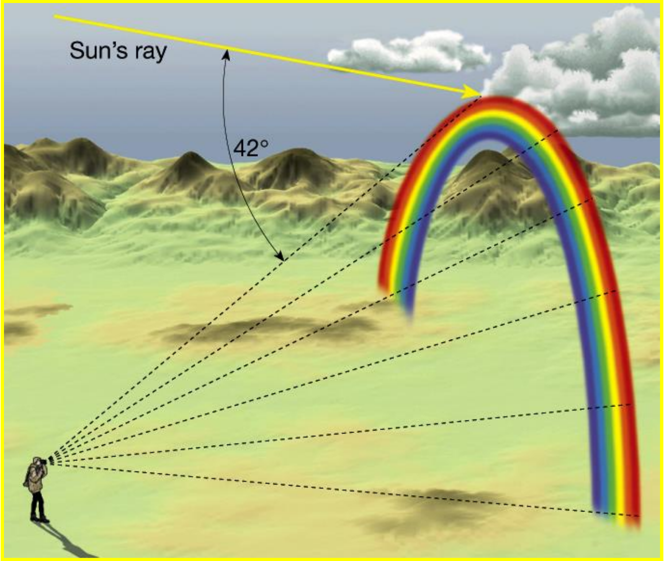




1. **Monochromatic**
2. **Directional**
3. **Coherent**

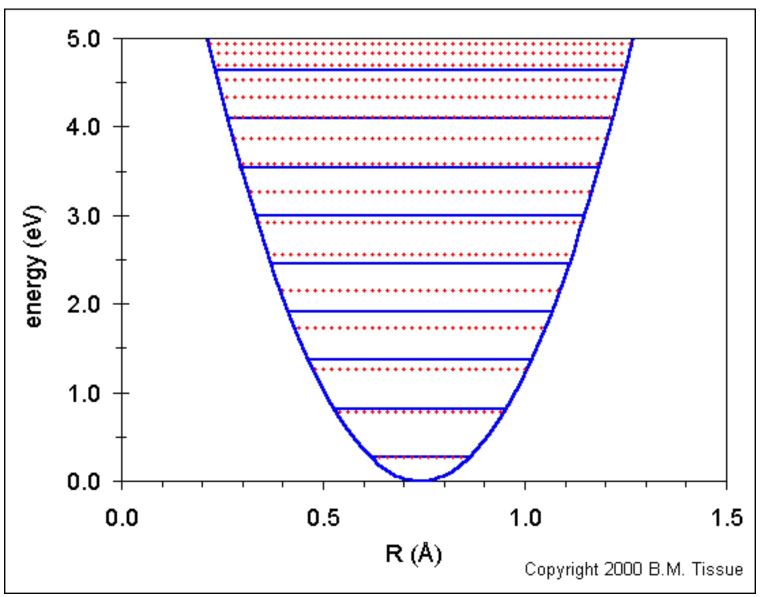
Light Phenomena - Nature





Linear Optics

- Interaction of weak electromagnetic field
- Harmonic Oscillator
- Phenomenon – Absorption, Refraction, Polarization



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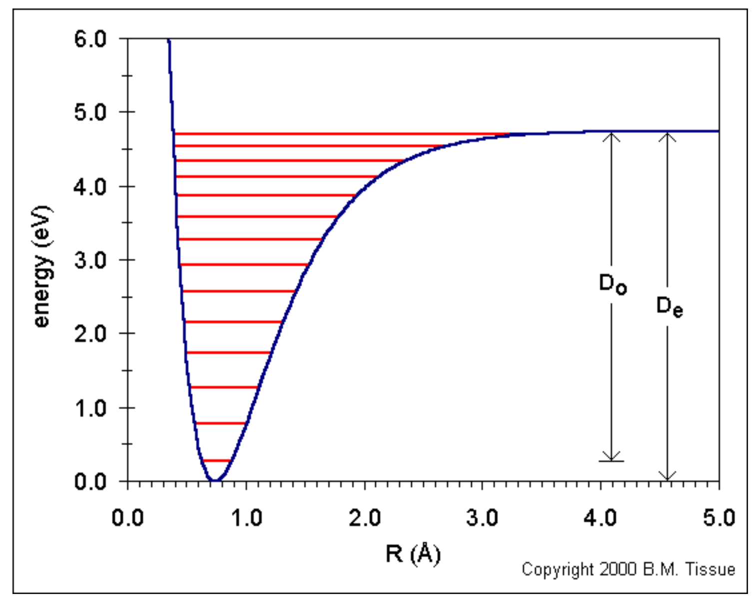
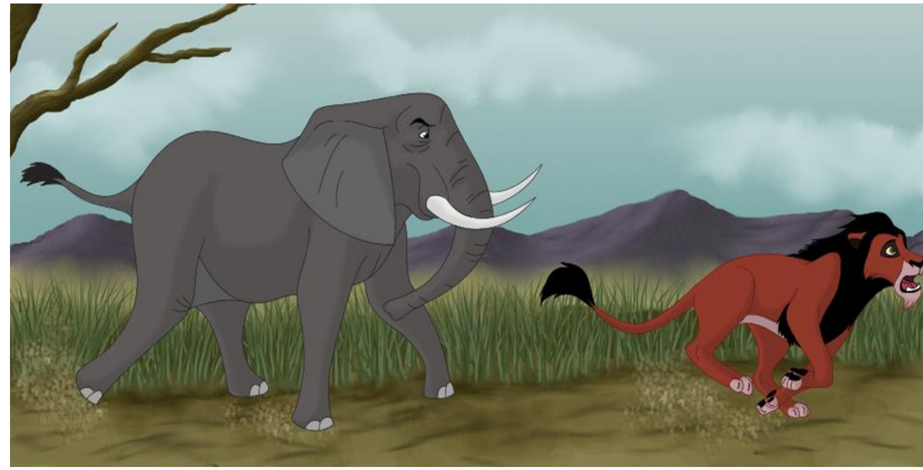
Nonlinear Optics

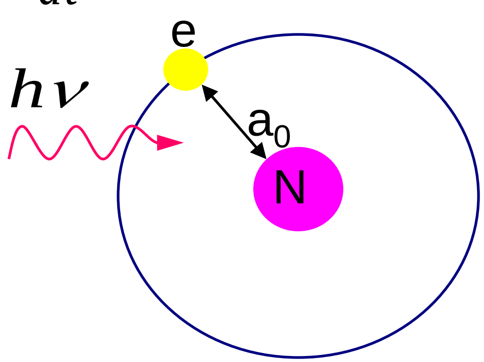
- Interaction of strong electromagnetic field
- Anharmonic Oscillator
- Phenomenon – nonlinear absorption, nonlinear refraction, nonlinear polarization



Origin of Nonlinear Optics



$$E_{at} \approx 2\times10^{+7}\,\text{esu}$$



$$P = \varepsilon_0\,\chi^{(1)}E + \varepsilon_0\,\chi^{(2)}EE + \varepsilon_0\,\chi^{(3)}EEE + \ldots$$

Its All About Maxwell Equations

The wave equation using Maxwell's equations $\nabla X \nabla X \boldsymbol{E} + \mu_0 \frac{\partial^2}{\partial t^2} \boldsymbol{D} = 0$

where $\boldsymbol{E}(r,t)$ is the electric field and $\boldsymbol{D}(r,t)$ is the displacement current.

With unidirectional property of laser (TEM_{00} mode), the equation of the electric field propagating primarily in one direction is written as

$$\boldsymbol{E}\,(\boldsymbol{r},t) = \hat{e}\; A(Z,t)\,\frac{\omega_0}{\omega(Z)}\, e^{\left[i\left(\frac{kr^2}{2q(Z)}kz - \tan\left(\frac{Z}{Z_0}\right)\omega t\right)\right]}$$

where $\hat{\mathbf{e}}$ is unit vector, A is wave amplitude as function of space and time, **k** is the wave vector and ω is circular frequency of the rapidly oscillating wave.

The material can be nonlinear under laser interaction and the fields ***D*** and ***E*** are related by

$$\boldsymbol{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

Thus the wave equation describing the electric field in the medium generated by the driving polarization can be expressed as

$$-\nabla^2 \boldsymbol{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = -\frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \boldsymbol{P}_L}{\partial t^2} \qquad \text{and} \qquad -\nabla^2 \boldsymbol{E} + \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = -\frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \boldsymbol{P}_{NL}}{\partial t^2}$$

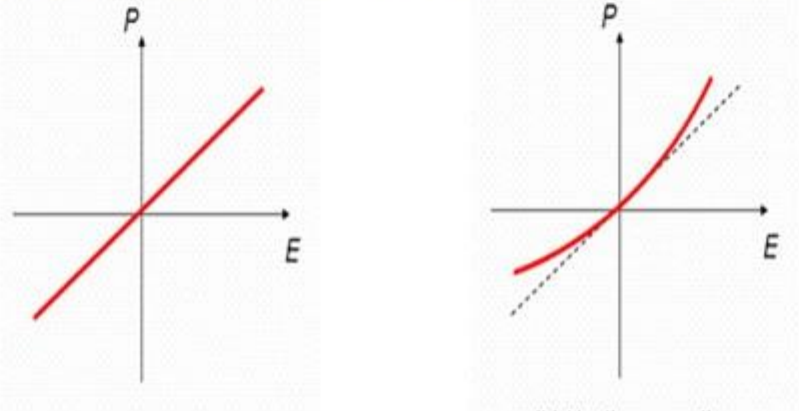
$$P_L(r,t) = \varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^{(1)}(r-r', t-t')\,\boldsymbol{E}\,(r',t')\,dr'\,dt'$$

$$P^{(2)}(r,t) = \varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^{(2)}(t-t', t-t'')\,\boldsymbol{E}\,(r,t')\,\boldsymbol{E}\,(r,t'')\,dt'\,dt''$$

Polarization – Linear and Nonlinear

Linear Polarization

$$\mathbf{P}^{(\mathrm{L})} = \varepsilon_0 \chi^{(1)} \boldsymbol{E}$$



(a) Linear dielectric medium (b) Nonlinear medium

Nonlinear Polarization

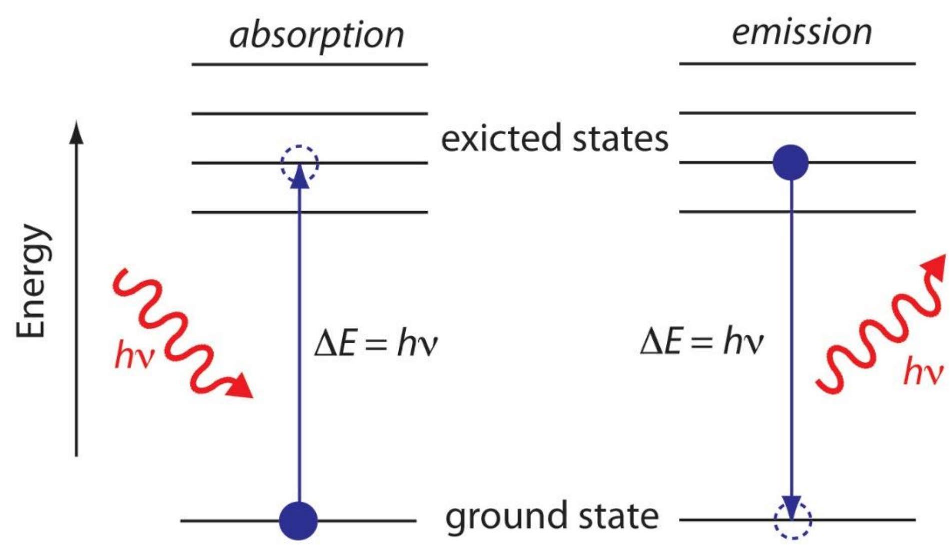
$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \boldsymbol{E} + \chi^{(2)} \boldsymbol{EE} + \chi^{(3)} \boldsymbol{EEE} + \cdots \right)$$

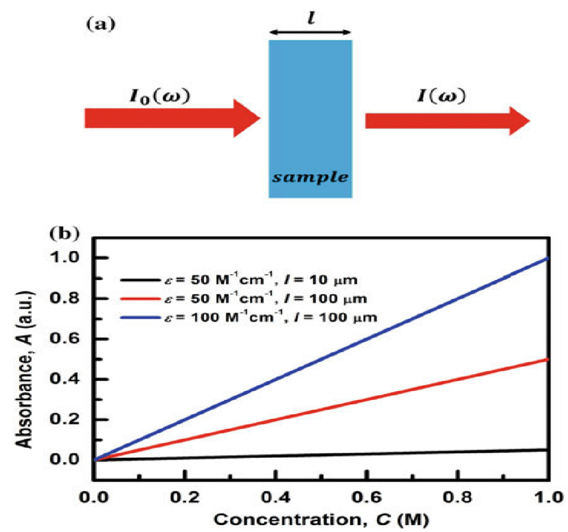
When $\mathbf{E} = \boldsymbol{E_0} \, \boldsymbol{cos}(\boldsymbol{\omega t})$

$$= \frac{1}{2} \varepsilon_0 \chi^{(2)} \mathrm{E}_0^2 + \varepsilon_0 \left(\chi^{(1)} + \frac{3}{4} \chi^{(3)} \mathrm{E}_0^2 \right) \mathrm{E}_0 \cos(\omega t)$$

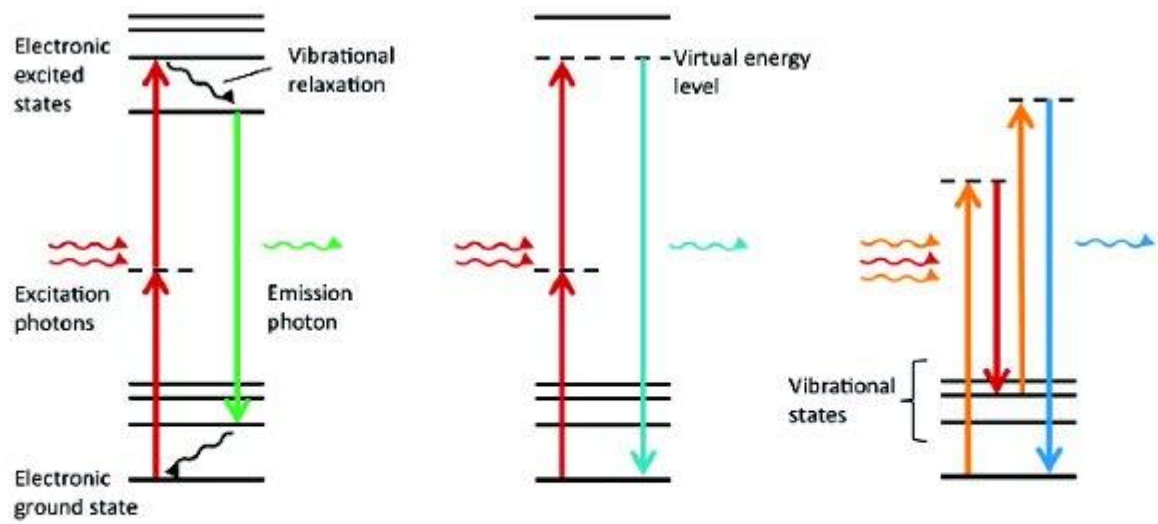
$$+ \frac{1}{2} \varepsilon_0 \chi^{(2)} \mathrm{E}_0^2 \cos(2\omega t) + \frac{1}{4} \varepsilon_0 \chi^{(3)} \mathrm{E}_0^3 \cos(3\omega t) \ldots$$

Absorption – Linear and Nonlinear





Linear Absorption



Nonlinear Absorption

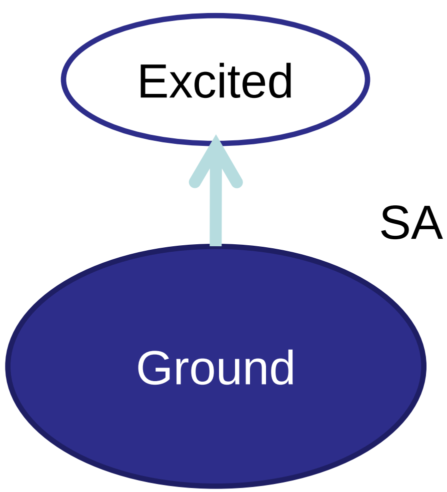
$$\alpha(I) = \alpha_0 + \beta I$$

Nonlinear Absorption

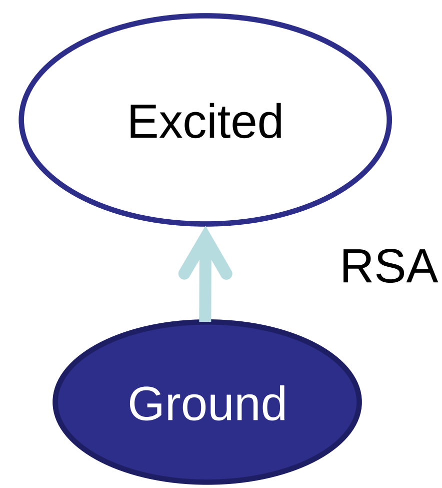
Change in transmittance of a material as a function of intensity

Absorption cross-section of the excited state < ground state – highly transmissive when excited - **saturable absorption** (SA).

Absorption cross-section of the excited state > ground state - less transmissive when excited - **reverse saturable absorption** (RSA).

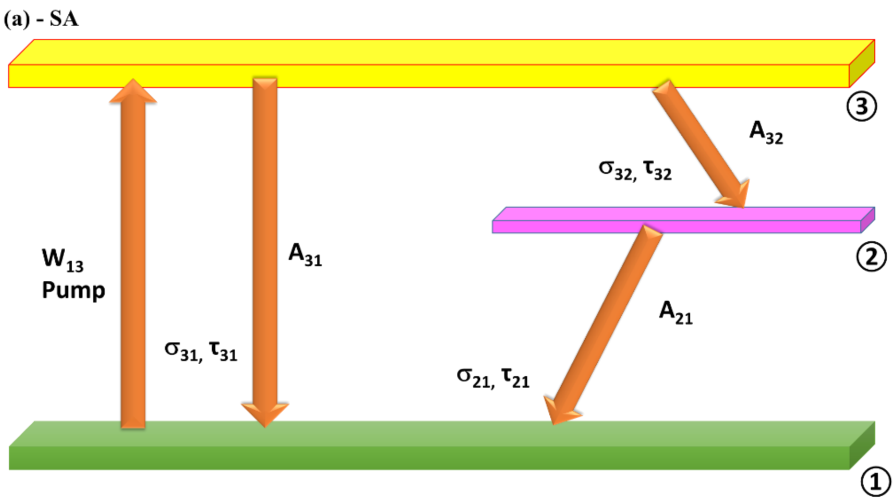


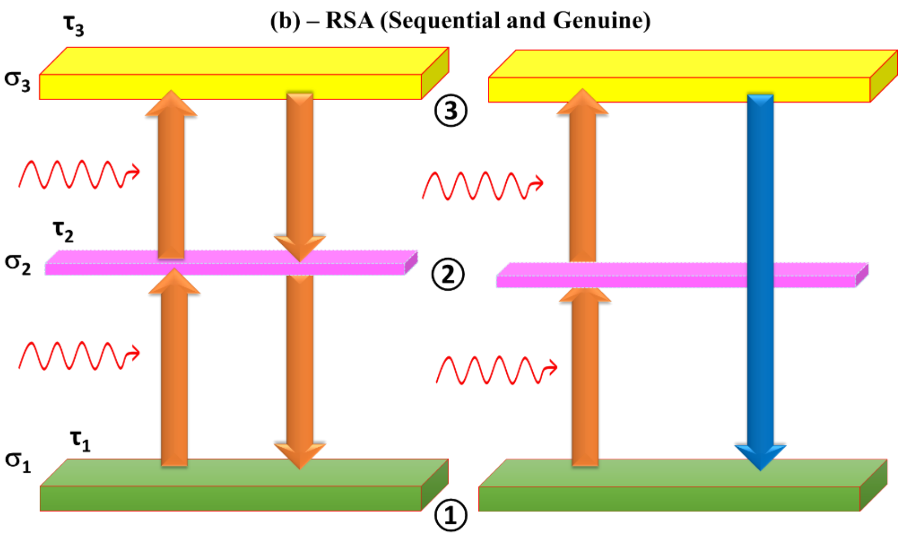
$$\alpha(I) = \alpha_0 + \beta I$$



Nonlinear Absorption

Change in transmittance of a material as a function of intensity

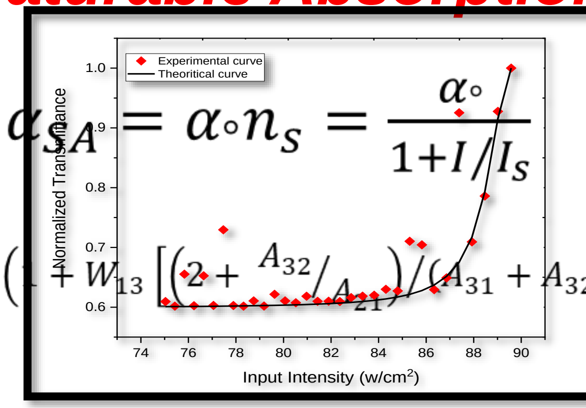




Saturable Absorption

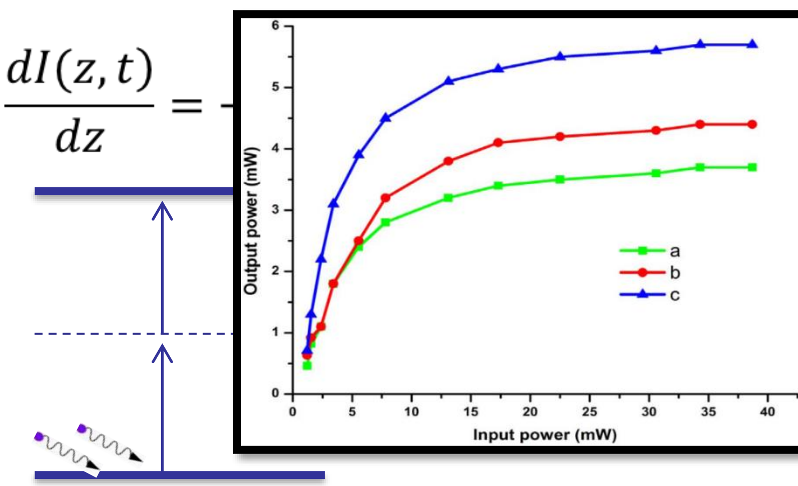
$$\alpha_{SA} = \alpha_\circ n_s = \frac{\alpha_\circ}{1+I/I_s}$$

$$n_s = \left(1 + W_{13}\left[\left(2 + A_{32}/A_{21}\right)/\left(A_{31}+A_{32}\right)\right]\right)^{-1}$$

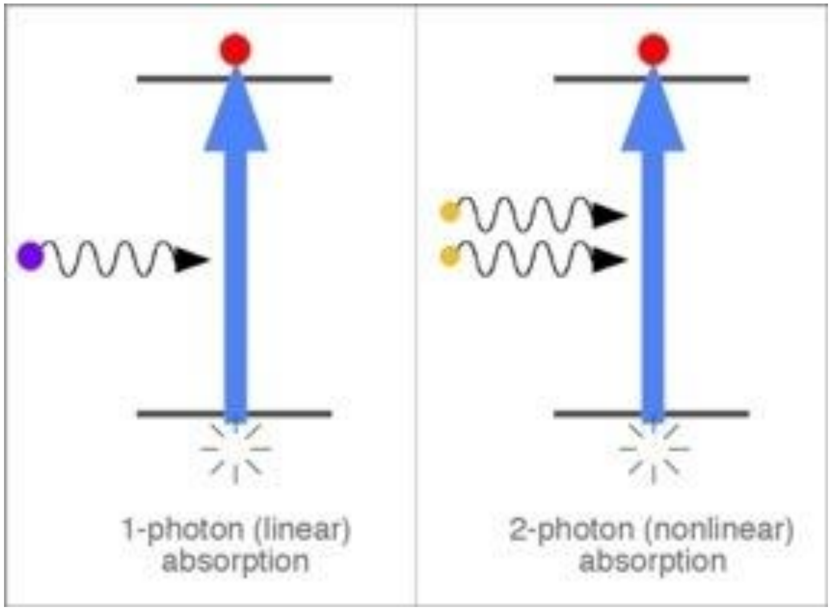


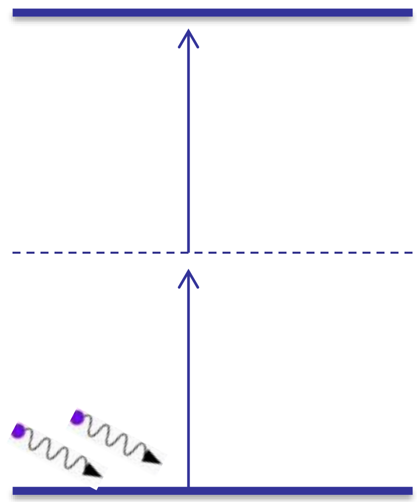
Reverse Saturable Absorption

$$\frac{dI(z,t)}{dz} = - \quad t) - \cdots,$$

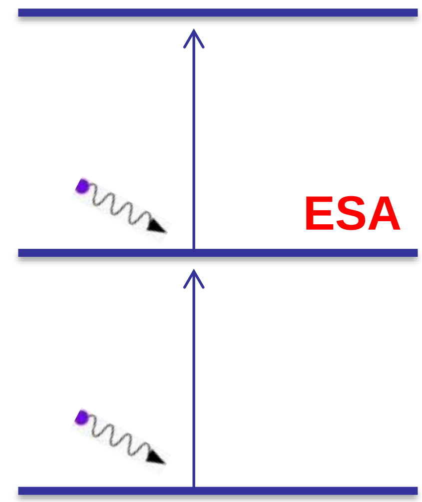


TWO PHOTON ABSORPTION



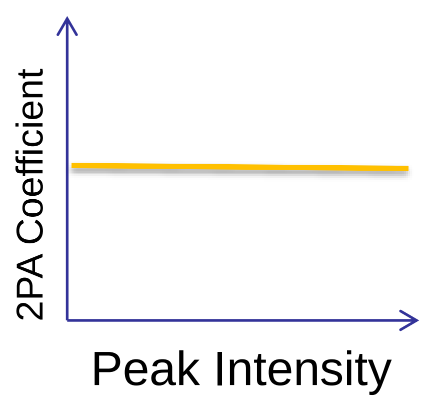


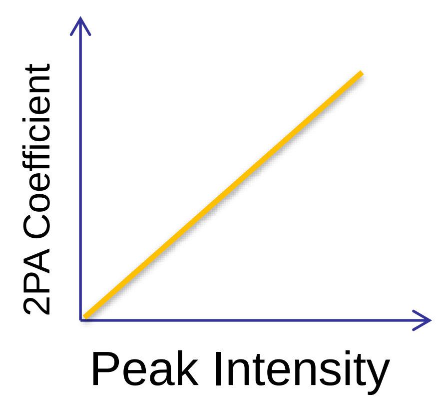
Geniune 2PA



Sequential 2PA

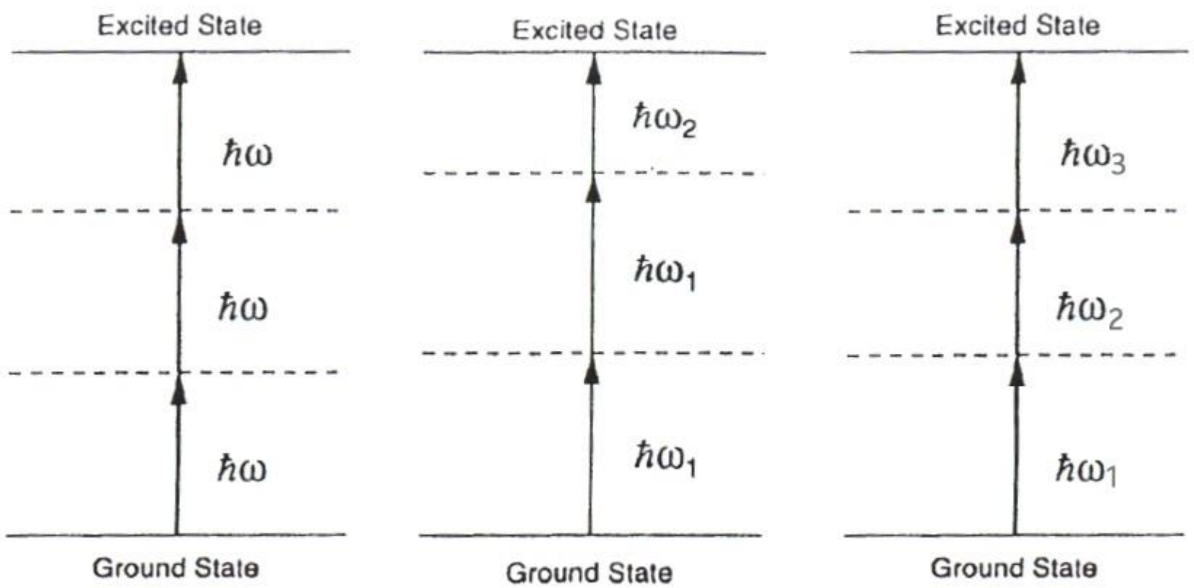
$$\frac{dI}{dz} = -\alpha I - \beta I^2$$





NLA - THREE PHOTON ABSORPTION

NLA-3PA (a) Self (b) Two Beam (c) Three beam



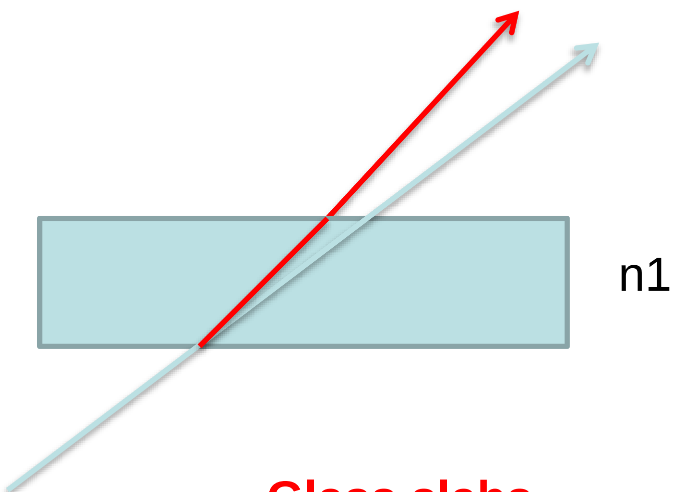
$$\frac{dI}{dz} = -\alpha I - \gamma I^3$$

Refraction – Linear and Nonlinear

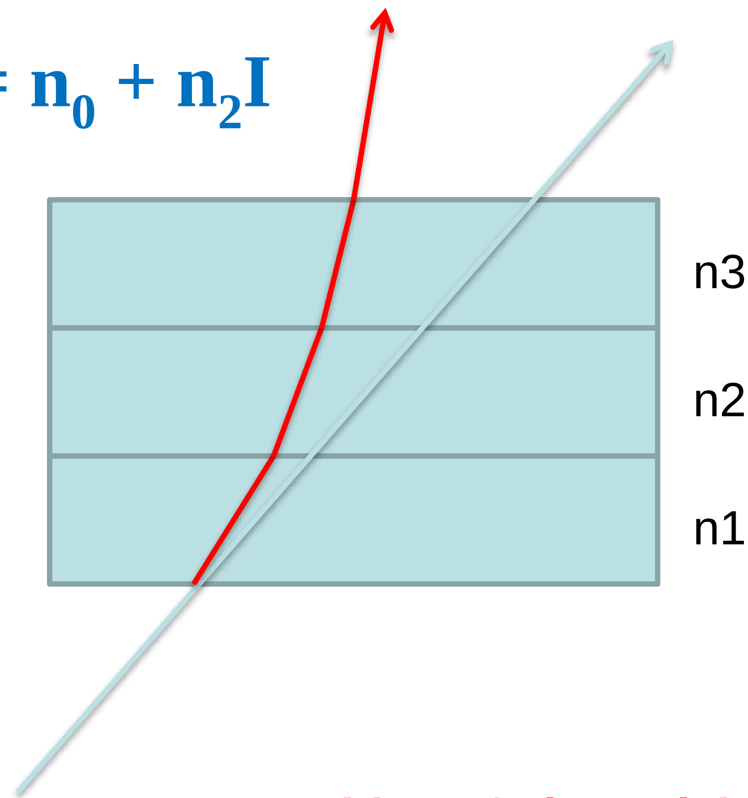
NONLINEAR REFRACTION

$$n(I) = n_0 + n_2 I$$

LINEAR REFRACTION



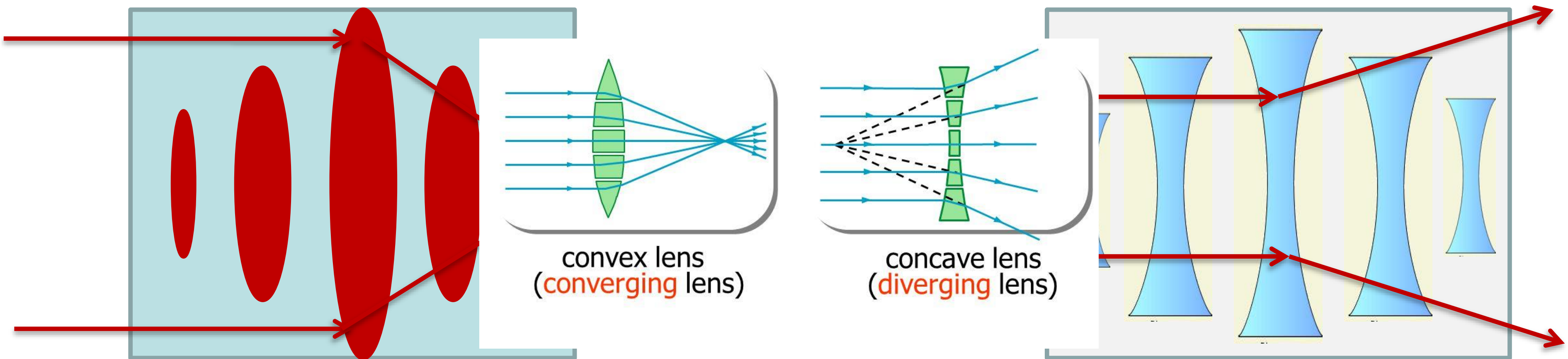
Glass slabs



Composition of glass slabs with different refractive index – acts like lens

Thermal Lens Model

- The temperature of the illuminated portion of the material consequently increases - change in the refractive index of the material - Thermal Lens effects.



$$n = n_0 + n_2 I$$

$n_2>0$ - Self-focusing

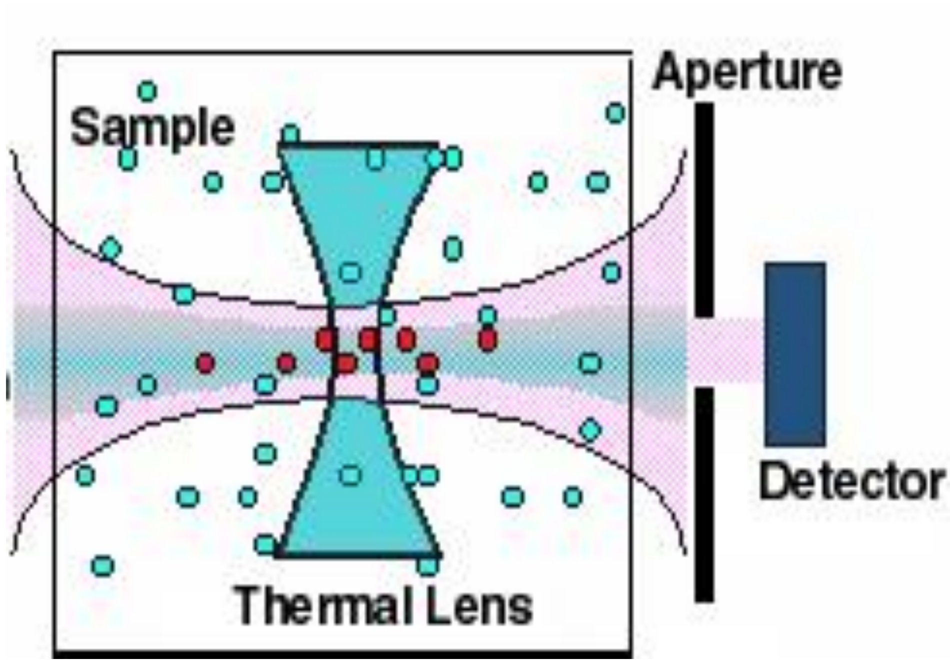
$n_2<0$ - Self-defocusing

For $n_2>0$ - beam collapse and phase distortion – Self-focusing – refractive index increases with intensity

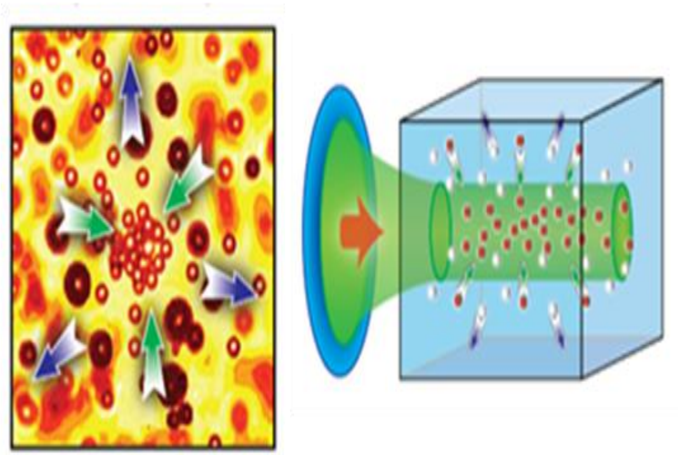
For $n_2<0$ - beam diverges more rapidly than collapsing – Self-defocusing – refractive index reduces with intensity

NONLINEAR REFRACTION - NLR

- Origin of thermal nonlinear optical effect - fraction of the incident laser power is absorbed - passing through an optical material.
- The temperature of the illuminated portion of the material consequently increases - change in the refractive index of the material.
- Thermal Lens effects - power / intensity in a continuous wave (cw) laser - pulse energy (fluence) in a pulsed laser.
- In the case of a cw laser beam, thermal effects are usually more dominant.

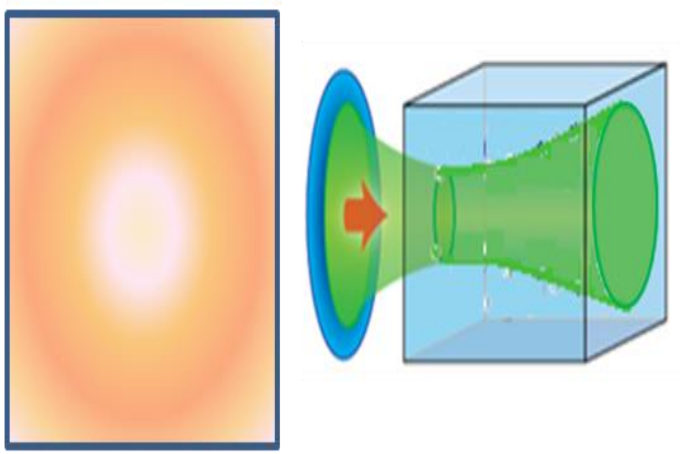


Self focusing and Self-Defocusing



$$\frac{dn}{dI} > 0$$

Self-focusing nonlinearity due to colloidal-particle redistribution driven by optical forces. Local refractive index n increases with light intensity I.



$$\frac{dn}{dI} < 0$$

Self-defocusing nonlinearity due to optical thermal effect in the m-cresol nylon solution. Refractive index n reduces with light intensity I .

Books for Study:

1. **K.R. Nambiar**, Lasers: *Principles, Types and Applications* (New Age Inter-national Publishers Ltd, New Delhi, 2014).
2. **B.B. Laud**, *Lasers and Nonlinear Optics,* 3rd Edn. (New Age International Pvt. Ltd., New Delhi, 2011).
3. **Ralf Menzel**, *Photonics* (Springer-Verlag Berlin Heidenberg, New York, 2007)

Books for Reference

1. **Richard L. Sutherland**, *Handbook of Nonlinear Optics*, (Marcel Decker Inc, New York, 2003)
2. **R.W. Boyd**, *Nonlinear Optics, 2nd* Edn. (Academic Press, New York, 2003)
3. **W.T. Silfvast**, *Laser Fundamentals* (Cambridge University Press, Cambridge, 2003)
4. **Y.R. Shen**, *The Principles of Nonlinear Optics*, (Wiley & Sons, New Jersey, 2003)